

Institution: Brunel University London
Unit of Assessment: 10 – Mathematical Sciences
<p>1. Unit context and structure, research and impact strategy</p> <p>Brunel University London is a research-intensive university which emphasizes academically rigorous research for addressing the challenges faced by a wide spectrum of end users, including governments, industry and the not-for-profit sector. The research strategy of the Department of Mathematics, and of the College of Engineering, Design and Physical Sciences (CEDPS) within which it sits, are fully aligned with the organisational culture of prioritising applicable and impactful research.</p> <p>Research within the Department has evolved and grown over the REF period, with a research-active headcount of 27 (26.4 FTE) compared to 26 (24.05 FTE) in 2014. Statistics and Data Science is identified as an area of strategic growth for the Department. Compared to REF 2014, 3.0 FTE additional staff are submitted for REF 2021 in this area.</p> <p>The Department's research is organized into four research groups, spanning a wide spectrum of sub-disciplines within mathematical sciences. Each of the research groups undertakes high quality research, embracing collaborations across disciplines and international boundaries and keeping sight of the needs of end users.</p> <p>The Applied and Numerical Analysis (ANA) group (Furter, Krasikov, Langdon, Maischak, Mikhailov, Shaw, Whiteman, Winter) carries out research both in the theoretical analysis of mathematical models of biological, chemical or physical processes described by differential, integral and integro-differential equations, and in the development, analysis and implementation of numerical methods for their approximate solution. The diverse applications considered by the group include rigorous analysis and computational modelling of problems in acoustic, elastic and electromagnetic wave propagation (Langdon, Maischak, Mikhailov, Shaw), modelling of fluid flows (Mikhailov), biological processes (Winter), viscoelasticity (Mikhailov, Shaw, Whiteman) and fracture in solids (Mikhailov). Novel finite element and boundary element methods have been developed and analysed for these applications, leading to both robust algorithms and specialised software. A related research topic is approximation of orthogonal polynomials and special functions with explicit error terms (Krasikov). ANA group research also includes analysis of nonlinear problems like the Navier-Stokes (Mikhailov) and Gierer-Meinhardt systems (Winter), and abstract bifurcation problems (Furter). The ANA group is home of an international collaboration on analytical and numerical methods for boundary-domain integral equations led by Prof. Mikhailov. This research direction, aimed at creating a new method for analysis and solution of variable-coefficient and nonlinear PDEs, was supported by several EPSRC and Royal Society research grants in the last 15 years, most recently by the EPSRC grant "Mathematical Analysis of Boundary-Domain Integral Equations for Nonlinear PDEs", £181K for 2015-2018. ANA group also hosts the triennial conference on Mathematics of Finite Elements and Applications (MAFELAP), which is one of the largest international conferences on this topic; both MAFELAP 2016 and MAFELAP 2019 during this REF period had over 350 delegates from more than 25 countries.</p> <p>The Mathematical Physics and Applied Mathematics (MPAM) group (Greenhow, Kaloghiros, Lawrie, Nolde, Pichugin, Rodgers, Savin, Smolyarenko, Virmani) is engaged in the development of innovative analytical methods for a broad range of problems arising from engineering and</p>

physical sciences. A particular specialism is the study of complex quantum systems, including quantum chaos, random matrix theory, quantum information and computing as well as research on related fundamental questions in geometry and mathematical physics. Significant contributions in the unit include the development of non-perturbative techniques for open wave chaotic systems (Savin), quantum computing and entanglement (Virmani), modelling and statistical analysis of complex networks (Rodgers, Smolyarenko), and geometry of (higher dimensional) algebraic varieties (Kaloghiros). Our international reach is evidenced by the annual series of Brunel-Bielefeld workshops on random matrix theory and applications, which is hosted on a rotational basis between Brunel and Bielefeld University in Germany, with each event in this REF period bringing together over 70 participants. The other strength of the group is physical modelling of wave phenomena in acoustic and elastic media. The relevant contributions include study of wave impacts on coastal structures (Greenhow), acoustic scattering in waveguides (Lawrie), asymptotic theory of thin elastic structures (Nolde, Pichugin), and layout optimization of structures (Pichugin). Dr Pichugin's research on layout optimisation for long span bridges has attracted a lot of media attention, including articles in the Times (22nd September 2018), New Civil Engineer (19th September 2018) and The Engineer (20th September 2018).

The **Financial Mathematics and Operational Research (FOR) group** (Boguslavskaya, Date, Lim, Lucas, Roman) develops and applies advanced mathematical, probabilistic and optimisation approaches to problems in Financial Mathematics (FM) and operational research (OR). Within FM, such problems include developing computationally efficient methods for pricing financial derivatives (Date, Lim), theoretical investigation into probabilistic interpretation of certain important financial models (Boguslavskaya, Lim) and optimisation of large financial portfolios under downside risk constraints (Date, Roman, Lucas). The recruitment of Dr Boguslavskaya and Dr Lim in 2018 has strengthened the Department's capabilities in probability and measure theory, which underpin financial mathematics. OR activities are driven by a range of applications which typically include simulation or optimisation in modelling and solving a complex decision-making problem. Contributions of the OR subgroup include modelling various financial portfolio optimisation problems under novel risk/return paradigms as integer or mixed integer programming problems, and development of metaheuristic solution methods for solving such problems (Date, Roman, Lucas). Two doctoral students in OR have been funded by industry partners in this REF period and several PhD alumni working in the UK finance industry continue to provide valuable end user inputs to research related to optimisation in financial applications and pricing of financial derivatives. More recently, research in the FOR group has also evolved to exploit the availability of high-volume news data, with innovative models developed to incorporate news analytics into prediction of prices and volatilities of financial assets (Date). Dr Date's paper on prediction of prices of crude oil futures contracts using macroeconomic news data received the IMA Journal of Management Mathematics Best Paper Prize in 2020. FOR group launched a revamped Financial Mathematics MSc programme in 2019-20, including modules on state-of-the-art research on machine learning and cryptocurrencies, offering a strong foundation for potential doctoral research candidates.

The **Statistics and Data Science (SDS) group** (Chakrabarty, Parker, Spire, Vinciotti, Yu) evolved from the Department's identification of data analytics and its applications as an area of strategic growth since 2015. The subject areas of Financial Mathematics, Operational Research and Statistics (FORS) formed a single research group in the 2007-2014 REF period. After the strategic decision to focus on data science as a growth area, the FORS group was re-organized into two distinct research groups in 2015: the FOR group mentioned above, and the SDS group, each of which has been strengthened through targeted recruitment during this REF period. With the recent appointments of Chakrabarty, Parker and Spire, the latter on a Royal Academy of

Engineering UK Intelligence Community Postdoctoral Research Fellowship, the SDS group has been able to place a particular emphasis on the strategic growth area of statistical methods and models for data science, with contributions in the field of experiment design, high-dimensional statistics and inference under censoring and missing data. The areas of applications of data analytics investigated by the group include astrophysics (Chakrabarty, Spire), biology and genomics (Parker, Yu, Vinciotti), financial econometrics (Vinciotti, Yu), health sciences and wellbeing (Vinciotti, Yu), material science (Chakrabarty, Yu), and UK intelligence (Chakrabarty, Spire). Funding for research and end user problems has come from industry (TWI, for analysis of pipeline corrosion), the EU (COSTNET project on network science), and the Office of National Statistics (for analyses of wellbeing data). There is also particular strength within the group in quantile regression and its application in bioscience, finance and other areas. A Statistics with Data Analytics MSc programme was started in 2017-18. The Department anticipates continued growth in this area in coming years, as it is developing a new undergraduate programme in Mathematics for Data Science, as part of a broader research and education collaboration with North China University of Technology (NCUT), to be delivered simultaneously in Brunel and in Beijing. This will further increase staff numbers and research strength in SDS in the coming years, with a view to putting the Department in a strong position for major cross-disciplinary funding applications where data science is a key enabling technology.

The departmental research strategy aims at facilitating and promoting world leading research, with particular emphasis on the mathematical formulation of fundamental problems inspired by real-world physical, engineering, financial and biological phenomena, and the development of associated advanced solution methodologies. Whilst the Department is keen on supporting and promoting high quality fundamental research, it actively encourages collaboration between its research groups, interdisciplinary research and the establishment of strong links with industry. These aims are closely aligned with College and University strategies which place a high value on engaging in innovative and cross-disciplinary research activities in line with the national agenda. As statistics and data science is identified as an area of strategic growth, the Department particularly encourages cross-disciplinary research which employs mathematical tools from data science to develop new theoretical, computational and methodological tools for simulating and solving important real-world problems, some of which are intractable by existing computational means. Examples include an ongoing collaboration between Dr Date (FOR) and Dr Shaw (ANA) on a novel use of a machine learning technique (otherwise employed in financial mathematics) for fast solutions of certain inverse problems in linear PDEs and a collaboration between Dr Smolyarenko (MPAM) and Dr Vinciotti (SDS) on new statistical tests for testing degree distributions in real-world networks.

The University also provides institutional support to cross-disciplinary research through the mechanism of research institutes. Many members of the Department belong to one of the four research institutes at Brunel: Dr Date belongs to the Institute of Energy Futures; Dr Maischak, Dr Shaw and Prof. Whiteman belong to the Institute of Materials and Manufacturing; Prof. Yu belongs to the Institute of Materials and Manufacturing, the Institute of Environment, Health and Societies and the Institute of Digital Futures; Dr Chakrabarty and Dr Parker belong to the Institute of Digital Futures. Research institutes facilitate collaboration across disciplinary boundaries; as an example, Prof. Yu's work on corrosion modelling in collaboration with Prof. Gan and Dr Wang from the Institute of Materials and Manufacturing has yielded important research contributions to corrosion defect assessment for the University's industrial partner TWI.

In this REF period, the members of staff contributing to this UoA have published, or had accepted for publication, 299 distinct refereed papers in over 140 leading journals. The 66

papers submitted for this UOA have a total (WoS) citation count of 410 (6.2 citations per paper). While citation data should be treated with caution, our high citation rates provide evidence to suggest that our research is setting the academic and intellectual agenda in relevant fields.

For effective global dissemination of our research Brunel has an Open Access mandate, and an Open Access Publishing Fund exists for supporting publication in open access journals. Members of staff thus have an excellent support network to draw upon for their research needs and ample means of global dissemination for their outputs. BURA (Brunel University Research Archive, <http://bura.brunel.ac.uk>), the University's open-access repository, holds copies of research outputs as well as doctoral theses and provides an effective means of dissemination. Digital data, metadata and/or code used to generate data used in research publications is held on the figshare repository (<https://brunel.figshare.com>).

In fact, the Department's commitment to open science goes further back than this REF period. In terms of the Department's research contribution to pedagogy of online teaching, Dr Greenhow from MPAM group made his *maths e.g* software for computer aided assessment in mathematics available globally in June 2011. As of December 2020, the current version of this software (<http://www.mathcentre.ac.uk:8081/mathseq/>) has more than 1100 unique teacher users across the world, in addition to a significantly higher number of student users. In the SDS group, Prof. Yu and his doctoral alumni have developed the R package *brq* (<https://cran.rproject.org/web/packages/Brq/Brq.pdf>), employed globally by researchers in Bayesian quantile regression. A software package, *maiprogs*, developed from research on numerical simulation of differential equations by Dr Maischak from the ANA group, is similarly freely available (<http://people.brunel.ac.uk/~mastmmm/doku.pdf>) and has been used by researchers worldwide, including research groups in Austria (Graz and Salzburg), Colombia (Medellin) and the U.K. (Edinburgh). The provision of accurate and reliable code and research data to allow for replication of published research results is part of a broader emphasis on research integrity in the Department. Brunel University London has adopted the Universities UK Concordat to support research integrity and is committed to upholding the essential elements of the concordat effectively and transparently. The Department adheres to the University's research integrity code and expects its staff and its research students to conduct their research in a professional and accountable manner which is consistent with the agreed code.

The impact strategy of the Department is built around identifying key industry partners and building long term research collaborations to solve a range of end user problems. As an example, Brunel has an extensive ongoing collaboration with TWI, Cambridge, in which the SDS group in the Department plays a very active role. A collaborative research project with TWI on pipeline corrosion analysis using quantile regression forms the basis of one of the unit's case studies. TWI and Prof. Yu from the SDS group have been working on industry relevant problems in reliability for several years and TWI has sponsored four PhD students in the Department during this REF period. Another longstanding industry partner is a local SME (Optirisk Systems UK) which has sponsored two doctoral research projects in the area of financial modelling and portfolio optimization during this REF period. It has also provided commercial economic data for some research projects. Previous research collaboration between Optirisk Systems and Dr Roman on application of second order stochastic dominance in financial portfolio optimization has resulted in development of software which is incorporated by Optirisk in a commercial product; further, Optirisk conducts training for industry practitioners which includes these research results. Optirisk is currently sponsoring one research student in the Department and is also providing news analytics data for another doctoral project. The Department is looking to

strengthen these ongoing collaborations and is seeking to build new industry partnerships in emerging research areas.

During the next five years the Department aims to expand the reach of the high quality research being undertaken by our existing research groups. Our ageing profile (25% of our academic staff is over the age of 60) is likely to provide the opportunity for renewal and reinvigoration of our research groups in the coming years, with an emphasis on ensuring a continued and increasing contribution to cutting edge topics in science and engineering. Current departmental strategy also includes plans to increase the population of doctoral students and the population of research assistants, and to maintain the current level of international collaborative research activity. The expansion in the number of RAs and doctoral students will be financed by an anticipated increase in interdisciplinary funding, which will be driven by carefully targeted applications to a range of funding bodies supporting the mathematical formulation and investigation of real-world problems. The Department is also expanding its international reach through collaborative agreements encompassing research and education, including NCUT (mentioned earlier), Anqing Normal University, China, and Maharaja Sayajirao University Baroda, India. These agreements will be used to build sustainable international research collaborations and will also provide an opportunity to recruit well-qualified doctoral students from partner universities.

The Department has a clear policy on balancing the workload between research and teaching and there is strong support for seeking external funding, including internal peer review arrangements. Staff who succeed in obtaining significant research funding are rewarded with a generous allowance of research time. The University and the College is also committed to supporting research in mathematics and the Department receives a share of EPSRC's Doctoral Training Partnership (DTP) grant from the University. In this REF period, five doctoral students who completed their doctoral degrees were funded by the DTP grant. As of December 2020, two students funded by DTP are continuing their doctoral studies in the Department. Further, a total of seven new doctoral students have started their research in the Department in the academic year 2020-21, with one more student expected to start in April 2021. This includes four highly qualified applicants supported by EPSRC studentships, through additional EPSRC funding for mathematics worth £107,000 and extra DTP funds made available by the University. In keeping with the Department's current strategy, one of these four studentships is funding a cross-disciplinary project combining our traditional strength in the MPAM group with the emerging area of data science, and addresses the use of machine learning for forecasting the behaviour of chaotic dynamical systems. Another studentship is within the SDS group, on a classical topic of Bayesian prediction problems in high dimensional data sets.

The departmental research strategy, which is in line with the University's research strategy of emphasizing academically rigorous and impactful research, is publicly accessible at <https://www.brunel.ac.uk/mathematics/research-and-phd-programmes>. The departmental research committee is responsible for monitoring its implementation and updating it annually.

2. People

i. Staffing strategy and staff development

The Department seeks to sustain and enhance its national and international profile in existing areas of strength, while responding to evolving end user demand by expanding into the strategic area of data science. To this end, it recruits staff who demonstrate research excellence that helps the Department to maintain its existing research strengths or strengthen the desired areas

of expansion, who have the potential to attract research funding and who can teach effectively at both UG and PG levels. Appointments at professorial/readership level are expected to have a strong track record in research leadership, obtaining research funding and supporting PhD students. All candidates are expected to have a plan for developing their research, establishing interdisciplinary collaborations and making funding applications. Every candidate for a faculty position is expected to present an overview of their research, and an interview panel then explores the candidate's future plans for research in greater detail.

As mentioned earlier, the Department has identified data science as a strategic growth area. Since 2019, two new members of staff have been recruited at senior lecturer level (Parker, Chakrabarty). The latter's appointment at Brunel has helped us to attract a Royal Academy of Engineering UK Intelligence Community Research Fellow (Spire). The Department also seeks to maintain its existing research strengths in FOR, ANA and MPAM research groups. It has strengthened its research in probability and measure theory as applied to financial mathematics by new recruitment at lecturer level (Boguslavskaya, Lim). In addition, Prof. Langdon joined as the Head of Department in 2019 and his research complements the existing research strengths of the ANA group.

More faculty recruitment is expected from 2021 onwards in the areas of mathematics that can contribute to research and teaching in the broad area of mathematics for data science, including statistics, data science, probability, operational research and numerical analysis. The Department will use the opportunity offered by anticipated growth in overseas student numbers, not least through our partnership with NCUT, to make strategic faculty recruitment, both to replenish our research groups in view of anticipated retirements, and to strengthen existing research activity across the department with a particular focus on growth of and interaction with the SDS group. The planned research and education partnership with NCUT will involve visits by faculty members to Beijing each year. This will be leveraged to enhance existing research collaborations in China, to build new collaborations around our existing research strengths and to attract high quality doctoral students. Other planned international collaborations in China (Anqing Normal University) and India (Maharaja Sayajirao University) are expected to bring similar benefits in terms of both deepening and broadening the Department's collaborative research activity, thereby strengthening its research capabilities and extending the reach of its research.

Recruiting, selecting, retaining and developing excellent researchers is one of the University's priorities. Brunel achieved the EU HR Excellence in Research award in 2011 in recognition of its alignment to the Researcher Development Concordat and its commitment to provide a supportive research environment for researchers. This award has been successfully reviewed every two years since, with the latest renewal being in 2020. The University has a publically accessible Concordat Action Plan

(<https://www.brunel.ac.uk/about/documents/pdf/BUL-Concordat-Action-Plan-2017-19-FINAL.pdf>) and a Concordat Working Group oversees its implementation.

Less experienced recruits, such as early career researchers, are assigned a senior academic as a mentor, and care is taken to ensure that all new staff are well integrated into the Department. A Staff Handbook detailing Departmental/University policies and procedures is provided and induction courses are run centrally by the University. In addition, the Department has a friendly and collegiate atmosphere which, coupled with the facilities offered by a departmental common room, ensures that ample opportunities exist for informal interaction and integration. Core skills

development and online research training is available to all contract research staff. In order to help them establish their research, newly appointed staff are assigned lighter teaching and administrative loads. Financial support for conferences/equipment is provided, and a wide range of development courses covering various aspects of research, teaching and administration are offered (attendance at some of these being obligatory). Less experienced recruits, such as early career researchers can also apply to a competitive university-wide Brunel Research Initiative and Enterprise Fund scheme for a centrally funded grant. Since 2014, two members of staff have been awarded a total of £29,500 under this scheme.

Support for career development of more established staff is directed towards ensuring that there are adequate resources available, in terms of research time, funding support, electronic resources and hardware. To ensure the availability of adequate time to do research, use is made of full- and part-time teaching-only staff in UG teaching roles. Research students are employed as paid Graduate Teaching Assistants (GTAs) to provide tutorial support at the UG level. Staff can also apply to the University-wide Research Leave Scheme and four members of staff have taken such leave between 2014 and 2019. As an example, Dr Date used his research leave in 2014-15 and an EPSRC overseas travel grant (EP/L019477/1) to visit an overseas collaborator, Dr Shovan Bhaumik at the Indian Institute of Technology Patna. The collaboration initiated during this period has been sustained through bilateral visits and it has led to several new results in filtering theory. Apart from a series of refereed journal publications, a graduate level textbook on filtering, co-authored by Dr Date and Dr Bhaumik, was published by CRC Press in 2019. Prof. Yu's research leave during this REF period was similarly productive, with several journal papers and one of the unit's impact case studies on pipeline corrosion resulting from his work during the research leave.

Great importance is attached to both staff and research students attending conferences in order to present their research and network with professional colleagues. All academic staff are provided with funding to attend at least one international conference per year and research contributions from the Department are presented at most major annual or biennial international conferences in the research areas represented in the Department, such as the EURO2015 - EURO2018 in FOR and CMStatistics 2015 - CMStatistics2019 in SDS.

As a part of its staff development strategy, the Department actively encourages exchange of ideas and research collaboration with industry, through longstanding and impactful research collaborations such as the one of Prof. Yu with TWI Cambridge mentioned earlier and also through engagement with its industrial advisory board. This advisory board, comprising experienced industry researchers and practitioners, offers advice both on end user priorities in applied research and on state-of-the-art content in educational programmes.

All lecturers/senior lecturers in the Department are appraised annually by a senior colleague, and Readers/Professors agree their research targets for the coming year with the Head of Department. Both systems of appraisal work in parallel with the departmental research strategy and yearly targets for individual staff are closely linked to the strategic aims. The annual performance management and appraisal processes are regularly reviewed and revised. Promotion procedures place a strong emphasis on international research excellence. In this REF period, eight members of the Department have been promoted, of whom four were women.

ii. Research students

The Department graduated a total of 39 (see REF 4a) doctoral students over the REF period, an average of 5.6 per year, which equates to 1.47 students per FTE staff member. Of these 39 students, 13 were women. The Department attracts a large number of national and international applications from potential research students of diverse ethnicities. The application process requires all prospective students to write a brief research plan which enables the Department to match students to supervisors with appropriate research interests. Candidates of appropriate academic calibre whose research interests align with the Department's research strategy are interviewed either in person or online by the prospective principal supervisor and one other academic, usually from the same research group. Apart from the competitive studentships facilitated by EPSRC funds as mentioned in Section 1, doctoral research is supported by our industrial partners (such as TWI and Optimisk Systems) and by governments or overseas educational institutions sponsoring their academic faculty for doctoral studies. Eight research students during this REF period were sponsored by universities in Saudi Arabia, Oman, Iraq and Malaysia. Research students from across the world, along with postdoctoral researchers and international visitors, have sustained a thriving research community in the Department throughout this REF period.

All PhD students are accommodated in shared offices, with a maximum of six students in each shared office. Student feedback suggests that this is a preferred model of working space, as compared to the alternative of having a single open plan space. The Department also has dedicated office space for academic visitors. Each research student and academic visitor gets a desk with a personal computer with essential software installed. Library and computing facilities are excellent: some software essential for research in mathematics is purchased from departmental funds and a wide range of relevant journals are available electronically, both on and off campus.

Brunel has made significant investment in providing high quality training and careers advice to research students and early career researchers. As mentioned earlier, the University's commitment to career development of researchers has been recognised by the EU HR excellence in research award, which was first received in 2011 and was renewed in 2020. The Department ensures that its PhD students and early career researchers get full benefit of the university provision of career development activities. Students are strongly encouraged to take advantage of the *Researcher Development Programme* which is organised on a university-wide basis by the Brunel Graduate School (BGS) and offers a comprehensive suite of personal and professional skills development (<http://www.brunel.ac.uk/services/graduate-school/training-development-and-support/researchstudents/researcher-development-programme>). A wide range of courses are offered including: new researcher induction (for all new PhD students); intermediate/advanced researcher development; research integrity; presentation skills; viva preparation; making the most of conferences and employability skills (covering entrepreneurship, interview skills, personality assessment). All workshops/seminars are focused towards the acquisition of the essential skills, attributes and behaviours highlighted in the *Researcher Development Framework*. In addition, Brunel Educational Excellence Centre runs a mandatory training course for research students who are expected to take up Graduate Teaching Assistant duties, which leads to Associate Fellowship of the Higher Education Academy. The University has a dedicated research careers advisor for PhD students and post-doctoral researchers. Apart from the BGS, the library provides research engagement training for students covering topics such as managing references and bibliographies.

In addition to the university-wide training outlined above, the Department offers a range of both technical and professional skills training for its doctoral students. FOR research students are normally required to attend National Taught Course Centre for Operational Research (NATCOR) courses or relevant London Taught Course Centre (LTCC) courses and attendance is fully funded from departmental resources. FOR and SDS students also attend appropriate modules from our own MSc programmes in Financial Mathematics and Statistics with Data Analytics. All other research students attend pertinent courses run by the LTCC and MMath modules in the Department, as directed by their supervisors. All research students are also expected to complete a postgraduate level module on *Research Methods and Case Studies*, run internally by the Department. This module includes talks by industry experts which are designed to stimulate end-user-oriented research. Further, the Department organises a number of research student symposia per year. These offer students the opportunity to present their research, discuss ideas and share best practice. Participation is strongly encouraged and doctoral students are required to make presentations both to support the development of appropriate personal/professional skills and in preparation for talks at conferences and other external fora.

All research students are allocated two supervisors and a research development advisor from the full-time academic staff. This allows them to draw directly on complementary academic skills and supports the spread of best practice. Students meet regularly with their supervisory team and at least eight such meetings per annum are formally recorded. This enables a student's progress to be closely monitored. Progression is a formal process with four major reviews: one after four weeks of enrolment and then after nine, 20 and 30 months from enrolment. The four-week review is to ensure full engagement of the student in the doctoral programme. Each subsequent review has clearly defined objectives and the student is guided towards completion through a structured and documented process. For the nine, 20 and 30 month reviews, the student submits thesis chapters and/or draft papers, which are appropriate for their expected stage of research. These documents form the basis for a review at the progress panel consisting of two independent members of staff, one of whom acts as a chair, and the student's second supervisor or research development advisor. The panel can also be attended by the first supervisor as an observer. Outcomes and planned deliverables since the previous review are assessed in each review and suitable new targets towards the next review (or the submission timeline) are agreed, as appropriate. The process is fully documented and, should the progress of any student provide a cause for concern, appropriate support mechanisms are put in place.

The programme of training as well as supervisory support provided for research students is monitored through regular meetings of the PGR Student-Staff Liaison Committee at College level and by the Director of Postgraduate Research. Feedback from doctoral students indicates a high level of satisfaction with the Department's support in their learning and progress in research. The success of our PhD programme is evidenced by the high number of PhD graduates attaining excellent careers in industry and academia. Recent examples include Dr Mukhamedov (2018) who is Head of Software Development at Minimax Labs UK, Dr Haselimashhadi (2017) who is Senior Data Scientist at European Bioinformatics Institute, U.K., Dr Islyayev (2015) who is a manager of Quantitative Advisory Service at EY UK, Dr Hakim (2014) who is a Lecturer in Mathematics at the University of Exeter and Dr Portillo (2016) who was a Senior Lecturer at Oxford Brookes University before recently moving to Loyola University in Spain. Besides, several doctoral students were employed by and funded by overseas universities and have secured career enhancement through their doctoral completion: Dr Allahyani (2018), Dr Alwohaibi (2018) and Dr Kalkatawi (2017) are all members of academic faculty at King Abdulla University in Saudi Arabia whereas Dr Alkenani (2013) is now a College

Dean at the University of Al-Qadisiyah, Iraq and Dr Hashem (2017) is a lecturer and Head of Department at the University of Duhok, Iraq.

iii. Equality and Diversity

Brunel, as a research-intensive university, is submitting 100% of all eligible academic staff. We followed the processes set out in our Code of Practice for the fair and transparent identification of independent researchers and to ensure that the submitted outputs provided a balanced and unbiased representation of the work of our diverse academic community, their characteristics and contractual positions (age, disability, ethnicity, gender, part-time workers and fixed-term employees). Output selection was monitored through regular Equality Impact Assessments. In 2012 Brunel received an Athena SWAN Bronze award for its efforts towards promoting equal opportunity for women in science-related subjects (of which Mathematics is a major part at Brunel), and this award was successfully renewed in 2017. Brunel was also recognized as one of the pioneering employers for shared parental leave by Working Families charity in 2015. The University ethos, as recognised through the accolades mentioned above, is strongly evident within the Department which is committed both to the development and support of staff in line with the University's Concordat and to equal opportunity for women. The Department's commitment to equality extends to all staff, from contract research staff, early career researchers and newly recruited staff to more established faculty members. Brunel's Equality Impact Assessment for the Department indicates that the outputs submitted are a well-balanced representation of the protected characteristics and contractual positions of staff. For example, our submission has: 11% staff of Asian ethnicity contributing 17% of the outputs; 30% female staff contributing 28% of the outputs and 11% staff aged 30-39 contributing 12% of the outputs.

The promotion process in the university is subjected to an annual Equality Impact Assessment, and the University organises workshops, led by the University Equality and Diversity Champion, to support all those intending to apply for promotion. The Department is a member of the LMS Good Practice Scheme. Further, a number of new initiatives are implemented in the Department to promote workplace equality and staff wellbeing:

1. A detailed workload allocation model is being employed which takes into account individual staff circumstances. Early career academics are given lighter teaching and administrative load to allow them to build a strong foundation for their careers as independent researchers. Dr Kaloghiros used reduced teaching load during her probationary years to invest more time in building a research reputation as well as building a strong case for funding her research, leading to an EPSRC first grant award (£91K, 2017-2019). With strong support from the Department, Dr Kaloghiros was also successful in winning the prestigious Emmy Noether Fellowship award from the London Mathematical Society in 2020, for supporting her research programme while dealing with significant caring responsibilities.
2. Days and times of staff meetings are varied to enable participation from those on part time contracts.
3. Flexible working arrangements are available for those with caring responsibilities. For example, for a large part of this REF period Dr Vinciotti worked from home from 3pm onwards on each working day due to childcare responsibilities.

The Department's commitment to equality and diversity extends beyond its internal boundaries and colleagues are encouraged to seek avenues for international outreach activity in research where mathematics is in disadvantaged position. This is evidenced in the large number of international visitors from the global south during this REF period, including 10 from China and five from Ethiopia. Further, in 2020 Prof. Mikhailov secured a grant from the London

Mathematical Society for mentoring research in mathematics in Ethiopia. Using this grant, Prof. Mikhailov will co-supervise doctoral students and mentor a research group at Adama Science and Technology University, Ethiopia.

3. Income, infrastructure and facilities

Research in the Department broadly consists of two overlapping strands: disciplinary research driven by intellectual curiosity and interdisciplinary research driven by end user needs, the latter typically involving collaborators from industry and from other academic disciplines. Both these strands underpin excellence in the quality of research outputs and in research impact. Accordingly, the Department's research income strategy focusses on securing appropriate resources for supporting both strands of research. Sources targeted for research income generation include initiating and nurturing research collaborations with industry, UKRI funding (including area-specific calls) for disciplinary research and funding from charities such as the London Mathematical Society and the Royal Society for international collaboration and networking events. There is a process of internal peer review in place to enable high quality applications. Research funding applications and success in income generation form an integral part of annual review of performance. The Department had significant successes in terms of funding for individual disciplinary research projects during this REF period, as evidenced by EPSRC research grants for Dr Kaloghiros (£91K, 2017-2019), Prof. Mikhailov (£181K, 2015-2018) and Dr Virmani (£188K, 2013-14). In terms of industrial research, Prof. Yu has been successful in winning industry funding for research on statistical tools for condition monitoring of mechanical structures, and his work on pipeline corrosion forms the basis of one of the Department's impact case studies. Colleagues in the Department have also secured a steady stream of grants for international collaborations and networking events such as workshops and conferences; notable grants include support from the London Mathematical Society for holding Random Matrix Theory workshops (2014, 2016, 2018 and 2020) and individual grants for significant overseas collaboration by Dr Date (EPSRC) and Dr Winter (London Mathematical Society). The total value of the research grants awarded (excluding the DTP grants) over this REF period was £1.32 million.

The Department's research is well supported by the University's infrastructure, which incorporates the graduate school (discussed earlier), the library and the *Research Support and Development Office (RSDO)*. The University library holds over 232,000 book titles and subscribes to approximately 109,500 unique journal titles, of which over 97% are available in electronic form. All major research journals relevant to the Department are freely available electronically (both on and off campus). These are supplemented by a print collection of back issues of journals. Within the entire University collection there are approximately 8600 books, 13,000 e-books and 4000 journals of direct relevance to the Department.

The university library provides dedicated space for postgraduate students and academic researchers including an IT room that is open 24 hours a day, 7 days a week. The library provides access to an impressive range of specialised databases including financial and statistical ones such as: Bankscope, Bloomberg, Datastream, FAME, IFS online and Reuters 3000Xtra. More generally, it offers access to, for example, the Web of Science (WoS), Scopus, BRAD and BURA. The university's research database, BRAD (Brunel Research Database), provides a comprehensive record of publications for all staff. BRAD automatically finds new publications from databases such as WoS/Scopus and updates staff research profiles/web pages on the University website.

University-wide support in negotiating contracts and planning/preparing research proposals and knowledge transfer partnerships is provided by RSDO. Further, through its e-Newsletters and tri-annual internally published research magazine, RSDO promotes funding opportunities and circulates Brunel research news and key achievements.

In addition to the university-wide facilities outlined above, all departmental staff and research students have a dedicated PC each, with internet access and shared access to a number of Laser printers. A range of appropriate software (e.g. C++, Latex, Microsoft Office) is provided as standard. Specialised packages are provided either as an individual licence or via a shared licensing system. The main statistical packages are S-PLUS and R whilst Mathematica and MATLAB (for which the University holds a site-wide licence) are extensively used by the applied mathematicians. Mathematical programming is facilitated by modelling languages AMPL and MPL, solvers Cplex, FortMP, FortSP and development environment AMPLdev. For computationally intensive applications, the Department also has a dedicated Nvidia GPU based high performance computing cluster, with 20 nodes and 6 graphics cards. Additionally, the College is investing a further £46,000 in enhancing the HPC capabilities in the Department, which will gain 8 more nodes and 2 new GPU cards for the computing cluster in 2021. This investment will assist the ongoing research in the Department on solving computationally intensive inverse problems in PDEs and in the use of machine learning for risk measurement in very high dimensional financial datasets. The Department also hosts the well-known *OR-library* website (<http://people.brunel.ac.uk/~mastijb/jeb/info.html>). This site holds test data for a wide variety of OR problems and is used by researchers world-wide. The full-time Departmental Computing Officer maintains the equipment and provides user support.

The research of each group is supported by the University's seminar series funding, which allows the Department to organize a series of seminars of outstanding researchers (from both academia and industry; from the U.K. as well as from abroad) for each group, reflecting their current research interests. In this REF period, the Department has been allocated £27,100 by the University for holding seminars. In addition, the Department uses its own funds for supporting occasional seminars throughout the year, e.g. for inviting eminent overseas academics to Brunel while they are visiting the U.K.

The Department moved to its new premises in 2017 where most of the faculty are located in individual offices on the same floor. The Department has a dedicated common room for informal research meetings, a seminar room and a dedicated office space for academic visitors. More than £50,000 has been spent on refurbishment of these premises.

4. Collaboration and contribution to the research base, economy and society

As mentioned in Section 1, the Department has nurtured key industry collaborations with TWI and Optirisk Systems UK, through PGR student sponsorships for projects addressing end user needs. These collaborations have brought multiple benefits to the U.K.'s research base, directly leading to one of the unit's impact case studies on pipeline corrosion and also providing several highly trained researchers now working in industry. Among PhD graduates from this REF period who were directly funded by industry, Dr Sadik (2018, funded by Optirisk Systems) works as a quantitative researcher at Optirisk Systems whereas Dr Arteche (2017, funded by TWI) is Senior Project Manager at Paraxel, Nottingham.

Apart from its direct collaboration with industry, the Department has made significant contribution to the U.K.'s research base through its participation in multi-university doctoral training courses. In keeping with its strategy to facilitate world leading research, the Department puts great emphasis on high quality training for doctoral students, and encourages its faculty to contribute to training for doctoral students both within Brunel and in the broader academic community. Within the U.K., the Department has been an active partner in multi-university doctoral student training centres such as NATCOR and the LTCC. Dr Date has contributed to a NATCOR course on Convex Optimization in 2014 and 2016 whereas members of MPAM and ANA (Savin, Smolyarenko, Furter, Maischak, Kaloghiros) have taught a total of 10 doctoral courses at the LTCC in this REF period.

The Department also has a strong record of building sustained international collaborations, especially with academics in developing countries. This collaborative activity is helped by the Department's truly international profile – the faculty represents 11 different nationalities including the U.K. In this REF period, the Department welcomed 39 different academic research visitors from all over the world. As mentioned earlier in section 2, a large proportion of visitors were from the global south, including 10 visitors from various universities in China. Collaborative research generated through these visits encompasses diverse topics, ranging from economic data analysis of tourism in China (Yu) to study of problems in boundary-domain integral equations (Mikhailov). Funding from these visits came from various sources, including the EU, the London Mathematical Society and the Royal Society. The extent and depth of academic influence of the Department is reflected in the authorship of publications; during this REF period, the members of this Department have published refereed articles involving a total of 61 co-authors based in 20 different countries including the U.K.

Apart from collaboration with overseas scholars and groups, the Department's international profile offers scope for conference organisation which, in turn, ensures that the Department's contribution to the wider research base is recognised across the international audience. The following major conferences have been hosted at Brunel in this REF period:

- The ANA group organizes the triennial international conference on Mathematics of Finite Elements and its Applications (MAFELAP) at Brunel, as they have been doing for nearly 50 years now. This conference had over 350 delegates from more than 25 countries, both in 2016 and 2019.
- The MPAM group co-organizes the annual Brunel-Bielefeld workshop on Random Matrix Theory and Applications, which is hosted at Brunel and Bielefeld (Germany) in alternate years. Each of the four workshops hosted at Brunel during this REF period (2014, 2016, 2018 and 2020) were partially funded by the London Mathematical Society and attracted around 70 participants from across the world.
- The FOR group organized an LMS funded invited lecture series on *Functional Calculus and Fractional Stochastic Calculus, including Rough-Paths, with Applications*, in summer 2020.
- The SDS group organized three major international workshops during this REF period: a workshop on Methods for Big Data in 2016, a workshop on Modern Statistical Methods for Health and Environment in 2017 and a workshop on Statistical Network Science in 2018. Each of the three workshops attracted 50-60 participants.

Besides hosting conferences at Brunel, various members of faculty contribute to program committees of significant academic conferences hosted elsewhere:

- Prof. Langdon is a co-organizer of a meeting on 'Scattering and Propagation of Waves: Theoretical and Computational Challenges', to be held at the University of Reading as soon as circumstances permit (originally scheduled for Summer 2020).
- Dr Kaloghiros is on the steering committee of the algebraic geometry seminar series since 2018. This series consists of two to three seminars per term across different universities in England, with each seminar attracting between 25 and 40 academic researchers. Dr Kaloghiros has won competitive funding from the London Mathematical Society to host these seminars in 2020-21. In addition, Dr Kaloghiros was a co-organizer of British Algebraic Geometry meeting at the University of Liverpool (2019). This meeting had around 120 participants and had a Fields medallist (Prof. Birkar) as an invited speaker. Dr Kaloghiros remains a co-organizer of the next meeting in September 2021 at Imperial College London.
- Prof. Mikhailov was a member of the steering committee for the International Conference on Integral Methods in Science and Engineering (IMSE), St. Petersburg, Russia (2020); Brighton, UK (2018); Padua, Italy (2016).
- Dr Smolyarenko is on the program committee for the 9th International Conference on Complex Networks and their Applications, Madrid, Spain (2020).
- Dr Savin was a co-organizer of the International Conference on Random Matrices and Determinantal Process, Center for International Research in Mathematics, Lumini, France (2017).
- Dr Date was a member of the program committee of the Symposium on Quantitative Finance and Risk Analysis, Korfu, Greece (2017).
- Dr Vinciotti was the program chair for IDA 2014 at Leuven, Belgium and was the organizer of the workshop on Network Science and its Applications, Isaac Newton Institute, Cambridge (2016).
- Prof. Yu was a member of the program committee for the first International Conference on Big Data and Applied Statistics, Beijing, China (2014).

The significance of research contributions of some of the colleagues in the Department have also been recognised by invitations for keynote or plenary lectures at major international conferences:

- Prof. Yu gave plenary lectures at the International Conference on Data Science, Machine Learning and Statistics, Van, Turkey (2019) and at the First International Virtual Conference on Statistics, University of Al Qadislyah, Iraq (2020).
- Dr Date gave an invited lecture at the International Webinar conference on *Leading Your Business through Economic Crisis*, MS University Baroda, India (2020).
- Prof. Mikhailov gave a plenary lecture at EUROMECH Colloquium 577, Stuttgart, Germany (2015).

The contribution of the Department to the international research base extends to assisting with evaluation of research funding applications, both globally and in the U.K.:

- Dr Virmani and Prof. Yu are both on ESF College of Experts. Dr Virmani is also a frequent member of Irish government outer board for research proposal evaluation.
- Dr Date is a proposal evaluator for the National Science and Engineering Research Council (Canada).
- Prof. Yu is a proposal evaluator for the Medical Research Council as well as the European Science Foundation.
- Dr Vinciotti is a member of the Swedish Research Council evaluation panel for mathematics.

- Prof. Langdon is a panel member for the Quarterly Journal of Mechanics and Applied Mathematics (QJMAM) Fund for Applied Mathematics.

In addition, Prof. Mikhailov and Dr Virmani are full members of the EPSRC peer review college, and Dr Boguslavskaya, Prof. Langdon, Dr Shaw and Prof. Yu are associate members of the EPSRC peer review college.

The Department also features prominently in terms of contributing to editorial work for scholarly international journals in mathematics.

- Prof. Yu and Dr Vinciotti are both Associate Editors of the *Journal of the Royal Statistical Society, series A*. Prof. Yu is also an Associate Editor of *Statistics and Its Interface*.
- Dr Date is an Associate Editor of *IMA Journal of Management Mathematics*.
- Dr Winter is an Associate Editor of *Abstract and Applied Analysis*.
- Prof. Mikhailov is an advisory Editor of the journal *Mathematical Methods in the Applied Sciences*.
- Prof. Whiteman has been joint Editor in Chief for *Numerical Methods for Partial Differential Equations* until December 2019 and is currently Emeritus Editor of this journal.
- Dr Shaw is a member of the editorial board for *Differential Equations and Dynamical Systems*.

Many members of staff have high h-indices, reflecting their lifelong impact on and contribution to the discipline. For the submitted faculty in this UoA, the average of WoS h-indices is 8.96, with 8 members of staff having h-indices of 13 or above. The international standing of some of the colleagues in the Department is also reflected in various fellowships from esteemed institutions:

- Prof. Yu is a Fellow of the Royal Statistical Society.
- Dr Date and Dr Shaw are both Fellows of the Institute of Mathematics and its Applications.