Institution: Royal Holloway University of London

Unit of Assessment: 10 Mathematical Sciences

1. Unit context and structure, research and impact strategy

1a Unit context and structure

Our B10 UoA submission is made up of the 19 Category A staff in the Mathematics Department.

Mathematics sits within the School of Engineering, Physical and Mathematical Sciences, alongside the Departments of Electronic Engineering, Computer Science, the Information Security Group (ISG) and Physics. An effective strategy for Mathematics has been to maintain a strong research focus in the areas listed below, enriched by a strong tradition of interdisciplinary activities and external engagement, especially with aspects of information security (it was previously paired with the Information Security Group as a School within the Science Faculty).

Mathematics hosts active research groups in three areas of strength:

- **Pure mathematics** (algebra, combinatorics and number theory, 10 FTE). Algebra focusses on various aspects of (both finite and infinite) group theory, with related areas such as representation theory, Lie algebras and algebraic topology also represented. Combinatorics includes graph and matroid theory, representations of the symmetric group, and applications of combinatorics to algorithm design, communication theory and cryptography. Number theory includes analytic, algebraic and combinatorial aspects.

- **Applied mathematics** (quantum dynamics, 6 FTE). We have strengths in quantum computation, quantum graphs, atoms in intense laser fields, and quantum foundations.

- **Statistics** (3 FTE). The group is active in probability and statistics, as well as mathematical finance, and applications in medicine.

The institution-wide transition to a School structure was made in 2019 (REF5a). Membership of the School of Engineering, Physical and Mathematical Sciences contributes positively to the research environment of this Unit of Assessment through increased cohesion and the exchange of best practice. A new research centre, **Combinatorial Methods in Algebra, Number Theory and Applications** (CANTA), was established in early 2020. With researchers across Mathematics, Computer Science, and Information Security, the CANTA provides a framework to develop cross-speciality and interdisciplinary synergies fully. The School structure has led the College to strategic investment in two new catalyst hubs, "Advanced Quantum Science and Technology" and "Transformational digital technologies, security and society", designed to strengthen industry-focused research, to which this unit will actively contribute. Our newly refurbished building, opened in 2020, allows the whole Mathematics department to be based in contiguous space and represents a major investment in our research environment. (Section 3c)

In REF2014 our overarching research aims were to (i) provide all staff with a supportive and stimulating environment, with the facilities and funding they need, so as to build upon the excellent mathematics research carried out at Royal Holloway in the groups listed above. We also aimed (ii) to recruit excellent staff to support these research groups as necessary.

Continuing and strengthening our REF2014 mission, we aim to: (a) **enable world-class research** in algebra, combinatorics, number theory, quantum dynamics and statistics and the links between them; (b) **encourage interdisciplinary work**, especially with Computer Science, Information Security and Physics, promoting impact; (c) to support **external engagement** with academics for community building, with user groups for impact, and with the general public to promote our enthusiasm for the subject; (d) form a **welcoming, positive and supportive environment** for all staff, students and visitors.

1b Research strategy and achievements 2014-2020

In REF2014, we described a research strategy for each group. A key measure for success during 2014-20 is the production of high-quality outputs, supporting our aims (a) and (b) above.

- **Algebra**. Our 2014 strategy included plans to make progress in infinite (often topologically motivated) group theory, and the representations of finite symmetric groups. Highlights of 2014-20 include: Barnea (Forum Math., Pi) has made major progress in the theory of subgroup growth of pro-p groups, answering a central question of Lubotzky and Segal. Kar (Comment. Math. Helv.) has completely characterised C*-simplicity for a group acting properly co-compactly on a finite dimensional CAT(0) cube complex. Nucinkis has uncovered unusual properties of irrational slope Thompson groups (Publ. Mat.) and has also progressed our understanding of conditions and examples of groups giving rise to nuclear semigroup C*-algebras (J. Functional Analysis). Wildon's Proc. LMS paper establishes the strongest results to date on general plethysms of Schur functions, making progress on a key problem of Stanley and proving several conjectures due to Agaoka.

- **Combinatorics**. Our 2014 strategy was to make progress in knot invariants for graph polynomials, and in interdisciplinary work described below. Moffatt (J. Comb Theory A) has established transformative results connecting delta-matroids with graphs embedded in topological spaces, allowing analogues of results on the Tutte polynomial to be proved for well-known polynomials associated with topological graphs.

- **Number Theory**. Our 2014 strategy emphasised research in probabilistic Galois theory, the theory of Salem and Pisot numbers, and the asymptotic counting of objects such as algebraic points of bounded height. To this end, Dietmann (Adv. Math.) has achieved a breakthrough in the probabilistic Galois theory of polynomials of degree 3 and 4, proving an old conjecture of van der Waerden for these degrees. McKee (J. Number Theory) settled a significant conjecture of Estes and Guralnik on minimal polynomials of rational symmetric matrices in the negative. Widmer (Math. Ann.) has established new upper bounds for the average l-torsion part of class groups over various families of number fields, and (IMRN) has established precise asymptotics for the number of n-tuples of algebraic integers generating number fields of fixed degree over a number field, ordered by absolute multiplicative Weil height.

- **Quantum Dynamics**. In 2014, our strategy called for progress in the links between quantum information theory and matrix analysis, in quantum graph theory, in atoms in an intense laser field, and in quantum foundations. Highlights include: Audenaert (Comm. Math. Phys.) has established sharp bounds on entanglement rates of evolving quantum spin systems, with important ramifications for the study of area laws in such systems. Bolte (J. Math. Phys.) uses quantum graphs to rigorously study Bose-Einstein condensation, and has completely classified such systems that exhibit Bose-Einstein condensation. Kay, as part as an EPSRC first grant, has constructed protocols that simplify the experimental requirements for quantum information and quantum computation. Mota-Furtado and O'Mahony (Phys. Rev. A) have reformulated the strong field approximation, the most prominent theory in the field. Schack is an author of a highly-cited paper (Amer. J. Physics) which has successfully established Quantum Bayesianism (now called QBism) as a mainstream interpretation of quantum mechanics.

- **Statistics** (which includes probability and mathematical finance). Our 2014 strategy planned work in stochastic approximation, interacting particle systems, probability in finance and applied areas. Highlights in the 2014-20 period include applied work described below and the work of Shcherbakov, who obtained strong results concerning the long-term behaviour of growth processes with graph-based interaction and competition processes (JAP, JSP, ALEA), and has also generalised the Black Scholes formula to pricing European options under the two-valued local volatility model.

- **Interdisciplinary and External Engagement**. The 2014 strategy promised progress on statistical applications to medicine, on random graph theory and algorithms, and on the mathematics of communication theory and complexity. A major highlight in the period 2014-20 was Koloydenko's prize-winning project on deep vein thrombosis. Sharia is involved with experimental design and the statistical analysis of clinical trials for a project investigating links between autism and eating disorders. Gerke (ACM Trans. Algorithms) has significantly improved understanding of important stochastic models of load balancing in parallel computations. Blackburn's work (ASIACRYPT,CRYPTO) on the cryptanalysis of group-based cryptosystems has influenced NIST Quantum Cryptography standards, and has led to an impact case study.

1c Research strategy 2021-26

We describe a strategy for each research group, supporting our aims (a) and (b), as follows:

- **Algebra.** The Algebra group plan to work in group theory and related areas. This includes: subgroup growth in groups and study groups acting on CAT(0)-spaces and their properties; continuing investigation of Thompson-like groups, with a new emphasis on studying links with Operator Algebra; continuing work in the plethysms of symmetric functions and the representation theory of the symmetric group, while branching out into exciting new areas in algebraic combinatorics.

- **Combinatorics**. The Combinatorics group will work on both pure and applied areas. Pure topics include the asymptotic enumeration of rings, and graph theory and matroid theory with a focus on topological structures, structural questions and algebraic invariants. Applied areas include applications to communication theory and post-quantum cryptography, and random graphs with real life networks in mind, considering resilience and equilibria questions.

- **Number Theory.** Planned research includes: exploring the interplay between Analytic Number Theory and Diophantine equations; attacking an old conjecture of van der Waerden on irreducible polynomials; working on the interface of number theory and combinatorics, studying Mahler measures along with associated combinatorial objects; studying problems related to class numbers in number fields, and geometry of numbers problems for certain Euclidean lattices of infinite rank.

- **Quantum Dynamics**. Research includes: studying asymptotic matrix inequalities with applications in quantum information theory; studies of quantum computation and technology, with primary focus on the theory of noise tolerance and reduction in modestly-sized computations; quantum foundations, concentrating on quantum Bayesianism and applications to quantum information theory; mathematical physics with a focus on many-particle quantum graphs and on semiclassical approximations for lattice Hamiltonians; studies of the interaction of intense short (10^{-16} seconds) laser pulses with atoms, through the theory of time dependent quantum systems, pursuing opportunities as members of the 'Attosecond Chemistry' COST network.

- **Statistics, probability and finance**. Research will include: working with biophysicists and medical partners to automate diagnosis and treatment of skin and breast cancers imaged with the help of Raman spectroscopy, as well as diagnosis and treatment of deep vein thrombosis imaged with various MRI modalities; work on Stochastic Approximation and collaborate with psychologists on efficacy of treatments for young people with anorexia nervosa; investigations of applications of probability to finance, and probabilistic models of multicomponent systems with interaction and their applications to modelling the spread of infection.

A further element of our future strategy, contributing to interdisciplinary research and growth of research income, is participation of the Mathematics UoA in two of the new College Catalysts. We will contribute to the catalyst in **Advanced Quantum Science and Technology** through the Quantum Dynamics group (building on expertise in theory of quantum computation, quantum theory and linear algebra), and through its interdisciplinary collaboration in post-quantum cryptography (Blackburn). We will contribute to the catalyst **Transformative Digital Technologies, Security and Society**, primarily through established and strong links between mathematics, theoretical computer science and information security, with new links promoted by CANTA.

1d Supporting interdisciplinary research and impact

Our approach to impact and interdisciplinary research (which are strongly linked in our discipline) supports aims (a), (b) and (c), and summarised as follows:

1. Our key strategy for industrial impact is to develop and maintain strong partnerships with external research users, particularly information security professionals. Working in close collaboration with the ISG, we participate in industrial liaison activities, such as the annual Hewlett-Packard Day for information security professionals, to maintain our business contacts. We supervise PhD students under the CDT in Cybersecurity.

2. Consultancy is encouraged with support from the College's Research & Innovation department, and the advice of our Director of Impact.

3. Secondments to the Heilbronn Institute. These have played a key role in delivering impact.

4. Public engagement is an important strand in our impact strategy, especially in pure areas, and contributes to the diversity of our subject area. We will maintain and support our Exploring Maths conference for 6th form students, our Coulter McDowell public lecture series, and our school talks programme.

5. We have pre-existing interdisciplinary links with Computer Science, ISG; CANTA will support and reinforce this work.

Evidence for the success of our industry strategy (approach 1) is provided by our highly rated impact case studies from REF2014 (2nd in the UK for impact). Our REF2021 impact case study on standards in post-quantum cryptography arose from this interdisciplinary framework and strong contacts in the information security industry, maintained by ISG. Our ongoing work on the next generation of banking standards for over-the-air card payments (initiated by Blackburn after attending an ISG research seminar) arose from collaboration with industry (1) and consultancy (2). Our confidential impact study arose from our secondment programme (3).

Selection of Impact Case Studies. An open invitation for proposals was made across the unit, facilitated by the Director of Impact who works with the Information Security Group, Computer Science and Mathematics. The confidential case was selected because of assurances of its intrinsic strength. The selection of the non-confidential case was made following internal and College-level review.

Evidence for the success of our interdisciplinary research (Approach 5) includes:

- 43% of PhD students supervised by Mathematics staff have a co-supervisor or advisor in a related department;
- the success of interdisciplinary grants such as the EPSRC-funded 'Private Information Retrieval with limited server storage: combinatorics and coding theory' (on the Maths-ISG borderline) and 'Synthesis of Quantum States' (Maths-Physics), and the meeting to celebrate Gregory Gutin's 60th birthday (Maths-CS);
- invitations to take part in Maths-CS and Maths-Physics EU COST networks.

Our **future impact strategy** will continue these approaches. It will also maximise the advantage from involvement in the two interdisciplinary Catalysts.

1e Research Integrity and an open research environment

Mathematics follows Royal Holloway's code for ethics, supported by the principles set out in the Concordat to Support Research Integrity, overseen by our institution's Research Ethics Committee, and supported by the Head of Department, the Doctoral School, and the School Research Committee. Ethical issues are discussed as part of PhD student annual review and in the (annual) periodic development review for staff members as appropriate. Expectations are set in the institution's PI Statement of Responsibilities for externally funded projects.

Our institution is committed to an open research environment, aligning with the four main UK HE funding bodies and the REF. Publications are deposited in a public institutional repository, unless there are special circumstances. Open access repositories such as arXiv, eCrypt and ResearchGate, and linked publications on homepages, are used for an overwhelming proportion of publications.

2. People

2a Staffing strategy

Our strategy is to maintain strength in our focus areas of research. For new or replacement staff positions, our policy is to strengthen our pre-existing research groups, build links between them and/or with interdisciplinary work, with an emphasis on research quality. Kar's appointment, strengthening the algebra group, demonstrates this policy in action.

Staff numbers have remained close to constant through the 2014-20 period, with a low turnover and sustainable demographic. There has been a net gain of 0.6 FTE in Mathematics since 2014, with the departure of Harman (0.2FTE, retired) and Klopsch (0.2FTE, moved to a full professorship in Germany) and the arrival of Kar (1.0FTE, from Southampton). All staff are on long-term contracts.

2b Staff development

All academic staff undertake an annual Personal Development Review with an experienced member of the department. Any appropriate institution-level training is identified: the excellent training in online communication technologies (e.g., research seminar delivery) has been particularly popular in 2020. This training is supplemented by discussions within the School and Department, a community wiki, and a virtual learning environment page for Maths staff to share their research. Funding is available for further professional development activities. The newly-formed CANTA research centre is taking a lead in subject-specific research training. The first event it organised was a virtual visit to the Department by the UKRI EPSRC Mathematical Sciences Team, to update on its vision, objectives and funding priorities.

The College's *Advance Programme* supports researchers at all career stages. Royal Holloway achieved the HR Excellence in Research Award in November 2019, demonstrating our commitment to the Concordat. The support for this Concordat at unit-level underpins the activities described in this section, reflecting our aim (d) from Section 1a, with measures aimed at creating a positive, supportive, fair, inclusive and vibrant research environment.

Mathematics organises regular events to provide opportunities for professional development, to reinforce our community, and to help integrate new staff. Our seminars and reading groups are part of our training and professional development, as well as adding value to the visits of external academics and spurring collaborations and communication. We have introduced regular symposium talks throughout the year for all mathematicians, and run weekly specialist seminars during term time, currently online. We organised an annual Statistics and Applied Probability Colloquium until 2018. We also host a series of Maths and ISG seminar talks for PhD students, organised by the students themselves. Staff and students regularly participate in reading groups in addition to these activities, examples in the REF period including groups discussing Chris Brookes' Part III course *Algebras*, Serre's *Trees*, a discussion of Classification Spaces, and a group that discusses recent papers in cryptography and information security. Other events include informal post-seminar refreshments, meetings discussing specialised topics (such as innovations in lecture delivery, and supporting CV writing), and an annual dinner after the Coulter McDowell lecture.

All staff have attended professional development activities offered by RHUL within the REF period. These activities include: Advance for research staff, Mandala for BAME staff, Springboard, Aurora Leadership and Women Enhancement Programmes for female research and related staff, Understanding Unconscious Bias, and GDPR training.

Probationary staff are given extra support. They benefit from the institution's Certificate in Academic Practice in Teaching and Learning (CAPITAL) programme, leading to a Higher Education Academy Fellowship; a direct application for a Fellowship is also possible. The department provides a significant workload reduction, to support development of a strong research programme. A probation advisor (a senior individual who does not line-manage the probationer) provides advice, encouragement and support. Early career researchers, including postdocs, have access to an ECR forum at School level, with representation on the School Research Committee, and an ECR programme of activities. Postdoctoral researchers have access to training programmes, and undertake Personal Development Reviews.

The Department provides funding to enable conference participation, underwrites grant applications to enable long-term planning of research visits and provides workload credit for the writing of significant applications. Staff have been successful in obtaining travel grants, conference grants, and grants supporting collaboration (e.g. EPSRC, LMS, Royal Society, Clay Mathematical Institute, Heilbronn Institute, European Commission). We have introduced a fund to support staff with caring responsibilities to enable attendance at short meetings.

College sabbatical policy has been applied to help staff finish a vital piece of work, or initiate a new research programme or project for impact or collaboration. Over the REF period, between 2 and 7 staff have taken up a one-term sabbatical each year. All but one staff member has been granted sabbatical time during the current period, and 25% of sabbatical terms were granted to female staff members during 2018-19 and 2019-20. Staff secondments promote impact and external engagement; 42 months of staff time, a significant proportion of staff time. Examples are: Koloydenko's sabbatical time, alongside equipment and travel funding, promoted his work on the automation of diagnosis and treatment of skin cancer imaged by Raman spectroscopy; Sharia's sabbatical, with departmental travel funding, supported her statistical analysis of randomised clinical trials in links between autism and eating disorders.

To recognise our staff members' achievements we have a criteria-based promotions policy, defined at institution level. External Engagement and Impact is one of the four categories that are assessed as part of the promotions process, in addition to categories for research, teaching and leadership. This has resulted in the following promotions during the current REF period: 5 promotions to Senior Lecturer (Kar, Kay, Koloydenko, Sharia, Shcherbakov), 4 promotions to Reader (Kay, Moffatt, Widmer, Wildon) and 9 promotions to Professor (Audenaert, Bolte, Dietmann, Gerke, Moffatt, Mota-Furtado, McKee, Widmer, Wildon).

2c Research students

We have a successful Doctoral School, attracting strong candidates from the UK and mainland Europe. We currently have 14 research students (21% female): 3 work in Algebra, 4 in Combinatorics, 3 in Number Theory, and 4 in Applied Mathematics. These numbers are similar to the REF2014 period.

To encourage a diverse field of applicants, we advertise PhD projects on findaphd.com, and on subject-specific fora such as number theory web, group pub forum and the IACR Open Positions in Cryptology list. All prospective PhD students are interviewed by a panel of at least two staff members in the discipline (who have completed unconscious bias training), to engage applicants with the department as a whole, and to ensure the applicant's interests and skills align well with their proposed research programme.

Currently 14% of our students are supported by the EPSRC-funded Centre for Doctoral Training in Cybersecurity. (Maths staff contributed to the bid for this funding and act as supervisors and mentors in this centre.) We have also hosted an EPSRC-funded CASE studentship, and obtained 54% FTE support from the EPSRC-funded UK Quantum Technology Hub for Quantum Communications Technologies. Over and above this funding, our students are of the calibre to obtain scholarships from our institution and other sources. Since 2014, 24 research students have finished their doctorates, and many have moved on to postdoctoral positions in the UK and abroad (Bilbao, Cambridge, Essex, Florence, Hagen, Southampton, Stuttgart, York). From earlier years, as evidence of long-term career development using skills developed on a PhD, we mention that number theorist Kaisa Matomäki (RHUL PhD 2009; now at Turku) was awarded a SASTRA Ramanujan Prize in 2016, the 2019 New Horizons Prize for Early-Career Achievement in Mathematics, and a European Mathematical Society Prize in 2020; Maura Paterson (RHUL PhD 2005) was promoted to a professorship at Birkbeck on the basis of her applicable combinatorics; Richard Horne (1995) is now Cyber Security Chair for Price Waterhouse Coopers.

Research students are an integral part of their respective research groups, participating in seminars, meetings and reading groups (see 2b). The Department is part of the London Taught Course Centre (LTCC), which provides courses that broaden our students' horizons. We support the LTCC by contributing financially and by delivering courses (such as an Advanced Course on Group Cohomology 2018 and 2019). We are organisers of, and a venue for, the Egham-Reading-London Arithmetic Statistics Seminars and the Functor Categories for Groups joint reseach group. Both are funded by the LMS under Scheme 3, which aims to enhance the education of beginning postgraduates and to support early career researchers. The Department provides generous funding for research students to attend national and international conferences, and encourages students to attend at least one conference per year of their study.

The Centre for Doctoral Training (CDT) in Cybersecurity provides extra opportunities for all our PhD students (whether or not CDT-sponsored): Students from Mathematics and the CDT hold joint seminars, and have a joint postgraduate conference every year, where students at the end of their first year present their research area. Those students on the CDT programme are obliged to take a range of courses designed to deepen their appreciation of the whole field of information security (which is much wider than mathematical subjects such as cryptography), and to undergo a 3-month placement with an external partner, usually from the information security industry.

The College Doctoral School runs a Researcher Development Programme, with a range of sessions including advice on CV preparation, job applications and career advice; the School runs a range of events that provide more science-focussed training. Since 2019, our students have access to the South East Physics graduate network (SEPNet), which provides a programme of advanced physics and professional skills training for PhD and early career researchers. So we provide a range of opportunities for students to widen their skills, and increase their contacts, across academia and industry as appropriate for their career aspirations.

Each PGR student is assigned a supervisor (sometimes two) and an adviser on arrival. They undergo an annual review to check progress and provide advice: a research report is produced, and the student presents an overview of their year to a committee consisting of their supervisor(s), advisor and an independent member of staff. All students formally arrive on an MPhil programme, and transfer to a full PhD programme after at most 20 months, on successful performance in an upgrade meeting.

2d Equality and Diversity

Equality and diversity issues are highlighted by Personal Development Reviews, by the School Equality and Diversity committee, and by our institution's policies. New departmental policies are agreed in our departmental meetings, open to all staff. We describe relevant aspects below, beginning with initiatives primarily relating to gender, but including wider issues of equality and diversity.

With over 25% female research active staff, the department is above the 2018/19 HESA London and SE average (of 20.4%). The number of women professors is three (also 25%, significantly above the 2018/19 average of 12.6%). RHUL offers promotions workshops for women academics, benefitting two Mathematicians in their successful applications for promotion.

Mathematics and ISG hold an Athena SWAN Bronze Award, after a successful submission in 2020. The process is more broadly supported by both Wildon and Moffatt sitting on AdvanceHE panels for Athena SWAN applications.

The department provides funding to, and strongly supports, the WISDOM group. Founded in May 2016 by two women, both PhD students, who wanted to raise the profile of women (staff and students), working and studying in the fields of Mathematics and Information Security, the scope of WISDOM has since expanded, stretching across multiple disciplines within the EPMS School and discussing equality and diversity in its broadest sense. WISDOM offers a strong support network for students and staff across the School who are interested in equality, and exploring ways that more women can enter and prosper within academia. Activities include monthly lunch meetings, workshops, discussion groups and outreach activities including socials. WISDOM's current goals, with particular emphasis on individuals from underrepresented groups, include: creating a welcoming and supportive environment, broadening the horizons and supporting career progression of researchers, raising the profile of research within the EPMS School and undertaking outreach efforts aimed at schools and undergraduates. WISDOM's achievements were recognised in 2017 with the Principal's award for enhancing fairness.

We are members of the London Mathematical Society (LMS) Good Practice Scheme to embed equal opportunities for women in our working practices. Wildon is a member of the RHUL Athena SWAN Champions Committee, and chairs the School Equality and Diversity committee. He convenes the School Equality and Diversity Reading Group. This promotes discussion and empowers staff and Ph.D. students to challenge sexism, racism and homophobia. One member commented 'It is eye opening, thought-provoking and challenges one's views of what a normal upbringing / life is and should be in the context of race, class, gender, diversity ...'

McKee has been a member of the LMS Women in Mathematics Committee, which promotes the diversity objectives of the Society. It was awarded the Royal Society's inaugural Athena prize in 2016 for 'introducing a broad range of initiatives in the field of mathematics that resulted in a change of culture that has happened nationwide, leading the way in increasing the number of women in mathematics.'

The department observed the 2020 #ShutdownSTEM initiative in support of black academic professionals. Nucinkis, as part of her work as chair of the LMS SLAM committee, has helped arrange (within the REF period) the Black Heroes of Mathematics meeting, which subsequently took place as part of the 2020 Black History Month (October 2020). All staff have attended training on unconscious bias, and many of the training programmes listed in Section 2b are relevant to equality and diversity. This training is, in particular, relevant to recruitment, and periodic developmental reviews. The department runs Equal Opportunity and Diversity training, attended by all new PhD students, which includes an introduction to unconscious bias, the nine protected characteristics and the College's support networks for LGBT+ and disabled students and staff.

One staff member was part-time from 2014-16; they have since returned to a full-time position, and have been promoted to professor. PhD students have the opportunity to mark course work; we have no other fixed-term staff. Support mechanisms and career pathways for students are described in Section 2c.

Measures to facilitate flexible working and to accommodate individuals' needs include: Timetabling teaching within core hours (10am to 4pm) if requested. Arranging department and school meetings and seminars within core hours. Encouraging remote working wherever possible and requested (now ubiquitous). Introducing a workload reduction of the equivalent of one term over three years for members of staff returning from extensive parental leave. Assisting staff returning after illness by reducing workload as needed. Making funding available for conference expenses arising from childcare, or due to ill-health. Helping staff with long-term illnesses or disabilities to maintain research productivity by accommodating their needs in a variety of ways, for example by ensuring that their workload is predictable and evenly spread out over the teaching terms. The department's workload model aims to provide a fair distribution of administrative and teaching work, to ensure that staff have equal time to develop their individual research programmes.

Our social and network events provide support that enhance staff and student wellbeing, and staff and students communicate informally outside these events. The College provides access to a 24-hour confidential Employee Wellbeing Programme provided by external provider Confidential Care, and has gathered an excellent selection of material to support wellbeing via Staff Wellbeing and Postgraduate Student Wellbeing webpages; this includes links to counselling services and working-well-from-home opportunities such as web-based social activities.

Equality and Diversity in the REF Submission: The department has followed our institution's code of practice for constructing its REF submission, as approved by Research England in August 2019 and revised in October 2020. The outputs were selected by a (33% female) committee of mathematicians from the department; each paper was reviewed internally by two reviewers, and external advice was sought on borderline cases and where the committee needed extra expertise; the output selection was approved by the college REF steering group. 30% of the final output selection was co-authored by women researchers (25% of our research active staff are women). Other REF documents were compiled by the department's REF Lead, and distributed to staff for accuracy and comment.

3. Income, infrastructure and facilities

3a Research Income

HESA data shows research income of £377k for the REF2021; £19.8k per staff FTE. This total does not include income from secondments, a significant contribution to the research impact of the department. Including this income (3.5 years at approximately £85k per year) brings our total income to £35.5k per staff member. There is also significant funding from other sources (such as funding received from EU grants where we are not the grant holder) that is also not included in the HESA figures; we provide more details below.

Small projects are the norm in our research areas. Our strategy is to concentrate on funding for meetings, small group collaboration and research networks, with EPSRC, the LMS, and the EU COST programme as our main sources. The aim is to nurture and extend research networks in our current areas of strength, and to explore neighbouring areas. All staff members explore potential avenues of research grant income as part of our annual appraisal cycle; the recent virtual visit by the EPSRC Mathematical Sciences Team has further encouraged this. Mathematics provides workload relief for writing large grant applications, and workload reorganisation to benefit the successful running of research grants. All research applications are peer-reviewed by another member of Mathematics or a sister department.

Highlights of our research income are listed below (where all EPSRC, LMS, Royal Society and EU funding was awarded on a competitive basis). We describe some of the high-quality outputs that arose from these activities, omitting co-authors.

- Approximately £300k income from secondment activity (which was used to partially fund the appointment of Kar, and will have produced impactful work), a major income for our unit. Two staff have undertaken secondments to the Heilbronn Institute, totalling 42 months.

- EPSRC funding: "Private Information Retrieval with limited server storage: combinatorics and coding theory" (£60k; Blackburn PI), funding research collaboration and a one-day meeting; "Synthesis of Quantum States" (£83k; Kay PI) funding research time. The former activity gave rise to widely cited papers by Blackburn (*IEEE Trans Inform. Theory* 2019, 2020) on private information retrieval; the latter produced papers on generating quantum spin states through spin chain dynamics (*New J. Phys* 2017) and on coprocessors for quantum devices (*Phys Rev A* 2018).

- Royal Society International Exchange: To collaborate with L. Reeves in Melbourne (£6k; Nucinkis). This led to two excellent papers on irrational slope versions of Thompson's groups by Nucinkis (Publ. Math, 2021, and arxiv 2020).

- LMS Scheme 3: Functor Categories for Groups (£4.5k; Nucinkis); ERLASS (the Egham-Reading-London Arithmetic Statistics Seminar) (£3k; Widmer). LMS Scheme 4 (Research in Pairs): visit to support collaboration with Barcelona and Zaragoza (£1k; Nucinkis), a collaboration with M. Menshikov (Shcherbakov; £0.4k), and a visit to RHUL of Jeffrey Vaaler (£1.2k; Widmer). LMS Scheme 6: Near Critical Systems: A workshop in celebration of M. Menshikov's 70th birthday (£4k; Shcherbakov). The LMS UK-Lebanon Atiyah Fellowship (Wildon; delayed due to Covid-19).

- Funding for a research meeting to celebrate Greg Gutin's 60th birthday (£6.5k; Blackburn and Gerke organisers).

- Our participation in the EU COST Networks IC1104 (Random Network Coding and Designs over GF(q); Blackburn working group chair; total budget €113k), CA18232 (Mathematical models for interacting dynamics on networks; Bolte), and CM1204 (XUV/X-ray light and fast ions for ultrafast chemistry; Mota-Furtado and O'Mahony; total budget €400k) which funded a significant number of meetings and research visits. IC1104 led to the EPSRC grant on Private Information Retrieval above; CM1204 led to papers in *Phys. Rev. A* in 2014 and 2016.

- Funding for meetings elsewhere: "Probabilistic Combinatorics: A Celebration of the work of Colin McDiarmid", Oxford (2016, Gerke); "Cohomological and Metric Properties of Groups of Homeomorphisms of R", Oberwolfach (2018, Nucinkis).

- Many externally-funded research visits, conference talks and research networks, including: Belgian WOG in Coding Theory and Cryptography (network; Blackburn; total budget €12k per year); RHUL-Technion research collaboration on applicable mathematics (Blackburn; £8k); Research in Pairs Oberwolfach (3 weeks; Bolte), seven-week visit to IME-USP, Sao Paulo (Scherbakov; £5k); visits to Victoria (CAN) and Wellington (NZ) (Nucinkis; approx. £4k); seven week Newton Institute visit (Nucinkis; £2k), invited talks at the British Combinatorial Conference (Gerke 2015; Moffatt 2019); and many more. The ongoing RHUL-Technion collaboration has led to progress in sequencing points of designs (since published in J. Comb. Designs, Blackburn, 2021); the Research in Pairs meeting led to progress in understanding a discrete variant of the Berry-Keating operator (Bolte, J. Phys A 2017); Nucinkis' visits to Victoria and Wellington produced work on the nuclearity of semigroup C*-algebras (JFA, 2021) and monoids in Thompson's group F (in preparation); Shcherbakov's Sao Paulo visit led to three papers on growth models, most notably in J. Stat. Phys. 2018.

3b Organisational infrastructure

There has been significant investment in research support in the School during this REF period, with a new full-time Research Administration Manager who manages post-award grant finances, provides administrative support for the organisation of funded meetings and advice on research planning and finances. The RHUL Research & Innovation Department provides support for contracts and the financial aspects of research applications, maintains research data via a research dashboard, provides training and networking opportunities for grant applications, and highlights relevant research calls to academic departments. The School Research and Knowledge Transfer Committee coordinates research strategy and supports both interdisciplinary activities, for example by promoting engagement of UoA with catalysts, and the promotion of impact.

3c Physical and operational infrastructure, and facilities

In a major investment by our institution, all unit staff are now in contiguous space in the newly refurbished McCrea Building. PhD students are located in the adjacent Bedford building, also completely refurbished, alongside students from Computer Science and ISG. Staff benefit from well-equipped individual offices. There are new spaces for both formal and informal interactions: coffee points, a discussion room dedicated to Maths, new printing facilities, a new display space in the foyer for outreach and celebrating our achievements, a space for post-seminar informal meetings, and new student contact points.

The department has a 4 FTE team of IT support staff, shared with Computer Science and ISG. The IT team supports and manages subject-specific computer labs (mainly used for teaching, but also available for training), and a dedicated GPU cluster. The team delivers day-to-day support to staff and PhD students, manages the department's software and hardware procurement, license management and renewals. In addition, the team manages installation and administration of desktop and server systems; configures and monitors our network infrastructure; researches, analyses and evaluates new and alternative technologies; provides and maintains a stable service, ensuring maximum availability; and supports specialist academic applications.

All staff and PhD students are entitled to a desktop or laptop every 4 years. In addition to this subject-specific support, the College's IT Services provides network infrastructure, general office software, and generic support. We have site licences for Mathematica, while subscriptions for MatLab and Magma are available on request. The institution-level Doctoral School provides admin support for PhD students, including monitoring annual review, and advertising training opportunities.

Modern lecture rooms, which include facilities for lecture capture and for live closed-captioning, are provided. A new £57m library and exhibition space, the Emily Wilding Davison Building, was opened in 2017 and houses study space for staff, students and visitors together with our main book and journal holdings. For 2019-20 (a typical year) our annual budget for books, shared with ISG, is £6.2k; standing orders for Mathematics journals is £74k, and in addition we subscribe to a wide range of journals produced by ACM, IEEE, and IOP. Our library is part of the JISC consortium for journal purchasing. All staff and PhD students have access to interlibrary loan facilities for books and articles with no maximum usage limit.

Equality and diversity issues relating to the time and confidence to develop research applications are addressed by the training and support networks, workload model and flexible working described in Section 2. Feedback from Personal Development Reviews (PDR) and Equality and Diversity Representatives indicates that there are no significant equality and diversity issues surrounding access to accessing scholarly or operational infrastructure in our department at present: our IT needs are well catered for, and 24-hour online access to library facilities allows flexible access practices. We encourage any issues to be brought up in PDRs, and commit to addressing any such issues if and when they arise.

4. Collaboration and contribution to the research base, economy and society

4a Collaboration within the discipline

The department has a thriving visitor and research visit programme, reflecting the importance of collaboration to most modern mathematics; 84% of our submitted outputs to REF2021 have co-authors outside the institution.

All our **academic visitors** have access to our IT and admin support. For short-term visitors guest logins are available, and we participate in Eduroam; longer-term or regular visitors are provided with full IT accounts which include printing and library access. Office space is provided when needed, most often an individual office of a member of staff on sabbatical. There are on-campus flats available, designed for academic visitors. A sample of the more regular or long-term visitors in this REF period include: Bradford (Goettingen), Balister (Memphis), Burillo (Barcelona), Chun (USA), Etzion (Technion), Frei (Manchester), Gross (USA), Martinez-Perez (Zaragoza), Lodha (Lausanne), Matucci (Campinas), Merino (Mexico), Mphako-Banda (South Africa), Reeves (Melbourne), Schlage-Puchta (Rostock), Stinson (Waterloo, Canada), Technau (Tel Aviv), Volkov (Sweden), and Zuk (Paris). Sebastian Egger visited to collaborate with Bolte as a Postdoc on a DFG-Postdoktorandenstipendium 02/14-08/15, and Zoltan Leka collaborated with Audenaert at RHUL on a two-year Marie-Curie fellowship "Moments", from September 2015 to August 2017.

Secondments to the Heilbronn Institute, totalling 42 months, have played a strategically important role, particularly on impact.

Departmental research funding of up to £2k per annum per researcher can be spent on **research visits**; many other visits are locally funded, or grant funded. The department grants sabbatical time for visits. Examples in the REF2021 period include Barcelona (Nucinkis), Bilbao (Kar), Bonn (Kar and Widmer), Fields Institute Toronto (Dietmann), Graz (Dietmann, Widmer), Louvain (Mota-Furtado, O'Mahony), Melbourne (Nucinkis), Sao Paulo (Shcherbakov), Singapore (Wildon), Stellenbosch (Schack), Stuttgart (Wildon), Vanderbilt (Kar), Victoria Canada (Nucinkis), Waterloo (Schack), Wellington NZ (Nucinkis), Zuerich (Gerke).

Research networks and partnerships

Involvement in **research networks** during the REF period, include three EU COST Actions, and two LMS Scheme 3 research networks: see Section 3a. Conferences and meetings are key to our research networks. Examples organised in the department include: Analysis on Graphs and Applications 2014 (Bolte), Workshop on New Directions for the Tutte Polynomial 2015 (Moffatt), Private Information Retrieval, Distributed Storage, and Network Coding 2016 (Blackburn), Near-Critical Stochastic Systems: A workshop in celebration of M. Menshikov's 70th birthday 2018 (Shcherbakov), Geometric Group Theory at RHUL 2019 (Nucinkis), two meetings on Functor Categories of Groups, and the RHUL Applied Statistics and Probability Colloquium. Activities we organised elsewhere include: two Dagstuhl Seminars on Graph Polynomials (Moffatt; 2016, 2019); a special session at the International Conference on Combinatorics, Graph Theory, and Computing, at Boca Raton (Moffatt; 2015), organising committee work for FPSAC '17, QMUL (Moffatt; 2017). the Young Geometric Group Theory at Oxford (Kar; 2017); an Oberwolfach workshop on Groups of homeomorphisms of the Real Line (Nucinkis; 2018).

4b Interdisciplinary work and national priorities

New initiatives to strengthen interdisciplinary activities include our involvement in two of the four college-level catalyst research and knowledge exchange hubs: the Digital Technologies, Cyber Security and Socioeconomic Impacts hub and the Advanced Quantum Science and Technologies hub. The former meshes with the EPSRC Digital Economy and ICT themes; the latter fits well with the EPSRC Quantum Technologies theme. These initiatives strengthen our already-close links with activities in information security and physics, which include consultancy projects (Gerke), joint research papers (Blackburn, Koloydenko), cross-disciplinary grants (Private Information Retrieval; Blackburn), and PhD supervision (Blackburn, Schack). Gerke is collaborating with Gutin (Computer Science) and Naery (Economics). Audenaert and Koloydenko are involved in long-term projects with CS and Mathematics (Nottingham), and Mathematics (Loughborough). The most recent (2010) International Review of Mathematical Sciences highlights the following opportunities to strengthen sub-areas, which we have been involved with: the links between combinatorics, graph theory and computer science (C-3); mathematical physics (C-8); number theory including links with cryptography (C-9); probability and links with other sciences (C-12).

4c Research for impact

Members of the Department are heavily involved in research impact, some already mentioned. Two members of the department were seconded for longer periods to the Heilbronn Institute for Mathematical Research, over two and three year periods respectively; a follow-up consultancy contract for up to 30 days work has been carried out for the same organisation. Some of this work has had classified impact. Blackburn's work is influencing the high-profle NIST Postquantum standard (one of our case studies) and on ISO standards for RFID in banking. Gerke carried out consultancy with Cyber First. Kay has released a LaTeX package called quantikz for creating quantum circuit diagrams, which is gaining popularity and has received a number of feature requests including a VP for Q Strategy and Ecosystem at IBM Research. Koloydenko is part of a large project *On automation of medical diagnosis and surgical treatment of skin and breast cancers based on Raman Spectroscopy* with Nottingham NHS Trust, Physics, and Psychology at the University of Nottingham, and with commercial and academic partners in Rotterdam (Netherlands). His project on deep vein thrombosis has won the 2019 Williams Prize of the Society of Academic and Research Surgery and Royal Society of Medicine; he has a Patent application as coinventor of the patents 'Measurement of Tissues Structures' (UK Application Number GB1305171.9) published 25/09/14, and publication number WO/2014/147416, published separately on 06/10/16 under US patent application number 14/778546. Sharia is collaborating with Prof. K. Tchanturia Institute of Psychiatry, Psychology and Neuroscience, King's College London on Pilot Randomized Controlled Trial of Cognitive Remediation Therapy in a specialist inpatient eating disorder services for children and adolescents.

Public Engagement

Impact through **public engagement** encourages diverse communities to take up Mathematics, and enhances the appreciation of the positive role of rigor in decision making and understanding. Every summer the Department hosts approximately 400 A-level students for its Exploring Mathematics Conference (55% female attendees) with guest speakers including Matt Parker, Simon Singh and Colin Wright. McKee has organised Pi Club, an added-value after-school club involving a consortium of local schools and hosted by the department. Our annual Coulter-McDowell lecture brings high-profile speakers (66% female since 2014) to the wider community. We publicise research highlights via news items on our home page, and via social media. Kar was awarded a British Science Association Media Fellowship 2019. Kay is the top-ranked contributor on the recently formed stack exchange website for quantum computing, where it is estimated that his answers have reached approximately 132k users. Barnea (top 7% in terms of users reached) and Wildon (top 3%) are significant contributors on MathOverflow.

4d Academic leadership and service to the academic community

The Department of Mathematics at RHUL has a strong tradition of service to the mathematical community, including involvement with learned societies, advisory bodies, editorships, guest lectureships and mini-courses.

Prizes: Koloydenko won the 2019 Williams Prize of the Society of Academic and Research Surgery and Royal Society of Medicine.

Learned Societies. Blackburn is an institutional representative of the London Mathematical Society (LMS), and served on its Nominating Committee in 2017. He is a member (and now treasurer) of the British Combinatorial Committee. Since 2019, he is a Senior member of the IEEE. McKee served on the LMS Women in Mathematics Committee. Moffatt transformed the

LMS Newsletter during his time as editor (2016-19), introducing a revised format, and encouraging a diverse set of contributors at all stages in their careers. Nucinkis was elected Member at Large of LMS Council in 2017. She serves on the LMS Committee for Society Lectures and Meetings (SLAM), which she chairs since November 2019. Schack is a Fellow of the Stellenbosch Institute for Advanced Study.

Invited talks at prestigious events include: the British Combinatorial Conference (Gerke 2015, Moffatt 2019); BMC/BAMC (Nucinkis 2015); workshops at Oberfolfach (Nucinkis 2015, Widmer 2016,2017, Wildon 2020), CIRM (Widmer 2014, 2018), Banff (McKee 2015), Max Plank Institute Dresden (Bolte 2015), and the Fields Institute (Nucinkis 2020); ICM Satellite events 2014 and 2018 (Dietmann, Nucinkis), plenary speakers at prestigious events such as the British Combinatorial Conference 2015 and 2019 (Gerke, Moffatt). Talks at the Fields Institute (Kay) and a Keynote at the European Congress of Mathematicians (Moffatt) are postponed due to Covid-19 but are due to take place in 2021. Many other invited talks were given: too many to list.

Editorial work. Blackburn has served on the editorial boards of J. Math. Cryptology; Cryptography and Communications; Groups, Complexity, Cryptology; and Mathematical Cryptology. Dietmann is an editorial board member for Ramanajan Journal, and McKee is an editorial board member for Mathematics of Computation. Moffat served as Guest editor of a special edition of Advances in Applied Mathematics (March 2018). Nucinkis edited an Oberwolfach Report (2019) and an Issue (444, 2017) of the LMS Lecture Notes (Proceedings of the 100th Durham Symposium).

Research Review. Blackburn was a sub-panel member for REF 2014, serving again for REF 2021, and is a Member of the EPSRC Peer Review College. Bolte and Schack are Advisory Board Members for J. Phys. A. We contribute reviews for the Leverhulme Trust, EPSRC, the AMS-NSA Mathematical Sciences Fund, FWF (Austrian Science Fund), NSERC (Canada), NCN (National Science Centre, Poland), the Czech Science Foundation, and ESF (European Science Fund).

Lectures and training for PGR students and early career researchers include the following. From September 2013 to August 2015 Audenaert was on secondment to the Department of Mathematical Physics, University of Ghent, Belgium. Invited lectures include: Kar at 'Groups and Geometries' Masterclass, Centre Internationale de Recontres Mathématiques-SMF, Marseilles; Schack at the Summer School on Complex Quantum Systems, University of Vienna (2014); Widmer at the Summer School "Analytic Number Theory and Diophantine Geometry", University of Hannover (2015). Nucinkis gave courses at the London Taught Couse Centre in 2018 and 2019. She was also invited to give a 20-hour graduate lecture course on Geometric Group theory, Fields Institute in Toronto in 2020 (cancelled due to the pandemic). PhD training is an explicit aim of our LMS Scheme 3 grants, and our EPSRC project in Private Information Retrieval.