Institution: Cardiff University

Unit of Assessment 10: Mathematical Sciences

1. Unit context and structure, research and impact strategy

1.1 Overview

Cardiff University School of Mathematics is an established research-intensive unit focused on advancing **fundamental mathematical knowledge**, **facilitating application of mathematical sciences in other disciplines, and providing societal benefit through engagement with industry, charities, and the public sector**. The School comprises 44 full-time teaching and research faculty, organised into five research groups: **Applied and Computational Mathematics**, led by Phillips; **Geometry, Algebra, math-Physics and Topology (GAPT)**, led initially by D.E. Evans, and now Lechner; **Mathematical Analysis**, led by Marletta; **Operational Research**, led by Harper; and **Statistics**, led by Zhigljavsky.

Focusing on critical mass, international reach, collaboration and a strong commitment to equality, diversity and inclusion, the School has developed considerably since REF2014. Major achievements of the period include:

- An increase of more than 50% in the total number of academic staff in the School, aligned to supporting five research groups of broadly equivalent size.
- Sustaining a fundamental ethos of academic freedom and pursuit of research for pure interest, which supported appointments from across the world and led to prestigious **international prizes for five colleagues** (Section 4.1).
- Securing University strategic investment for a new purpose-designed facility (the £39m Abacws building), now approaching completion, accelerating collaboration with Computer Science and Informatics.
- **Sustaining healthy support for PhD research, with a revised University approach to ESPRC-DTP studentship funding comparable to the very best we received under direct allocation from EPSRC, additionally opening up new opportunities for cross-disciplinary collaboration via Interdisciplinary Training Hubs (REF5a).**
- Improvements to PhD recruitment practices that successfully redressed gender imbalance, reaching a 54%-46% female/male enrolment in 2019/20, consistent with our fundamental commitment to equality, diversity and inclusion (Section 2.2, Fig. 1).
- A **vibrant** and **substantial programme of world-leading visitors and speakers. This included five Fields Medalists and more than three dozen winners of international prizes, including ICM keynote speakers (Table 4 in Section 4).**
- A new formal strategic partnership with the Office for National Statistics (ONS), including a Senior Lectureship in Statistics jointly funded with ONS.
- Societal benefits with significant international reach, including growing mathematical communities (in Africa), increased business effectiveness (Crimtan plc) and improved clinical services (for the NHS), as evidenced through our impact cases.
- Major new interdisciplinary collaborations including joint research in mathematical biology with the Wolfson Centre for Mathematical Biology in Oxford, and a €6.7M grant with the School of Earth and Environmental Sciences, Cardiff to address climate change resistance.

1.2 Progress since REF2014

Following REF2014, the School made a case to the University for a major expansion in Mathematics, which was supported as a top priority through our College of Physical Sciences and Engineering. Consequently, we appointed more than 20 new teaching and research staff (Table 1) over the REF period. Allowing for turnover, our FTE count has increased from 28 to 44. Aided by our External Advisory Board (Section 2.1) we drew up appointment strategies which balanced the competing desires for more investment in pure mathematics, including analysis, algebra and topology; expansion of the Operational Research Group beyond its healthcare focus; growth in

applied mathematics oriented towards biosciences and engineering; support of strong demand from across the University for collaboration with our statisticians; and the imperative to grow engagement with research users.

Furthermore, given the importance of collaboration with computer scientists in areas involving combinatorics, optimisation, data analysis and machine learning, we made a strategic decision to formalise existing links with Computer Science, including academics from each School sitting on appointment panels in the other. The two schools made a case to the University to invest £39m in a new purpose-designed 9700m^2 building, and will be co-located from autumn 2021 (Section 3.3). These formal links have already resulted in enhanced collaboration with strategic partners such as the ONS, which committed to funding a Senior Lectureship (Section 4.2.2). Further evidence of collaboration includes the joint directorship of the University's flagship Data Innovation Research Institute, led by Harper (Mathematics) and Spasić (Computer Science). At the same time, we expanded coverage of topics in algebra and geometry, in probability and in stochastic analysis, in kinetic theory and PDEs, fundamental areas aligned to facilitating shared future research directions across groups.

Group	New people	Current FTE
Mathematical Analysis	Ben-Artzi, Scheuer, Strokorb (0.5)	10.2
Computational & Applied	Dong, Kadri, Kaouri, Woolley	8.5
GAPT	Ros Camacho, Lechner, Naaijkens, Pennig, Wood	8
Statistics	Artemiou, Gauthier, Strokorb (0.5)	6.5
OR	Chen, Gagarin, Gartner, Jones, Liu, Oertel, Palmer	11

Table 1: New Teaching and Research staff since 2014

We decided soon after REF2014 to further emphasise dissemination and international visibility. We encouraged our researchers to propose and direct workshops at leading research institutes throughout the world (Section 4.4), supported by study leave whenever possible. We also decided to grow our visitor programme, see Table 4 in Section 4.1, with particular emphasis on research excellence. We believe that hosting eminent visitors helped raise our global presence and support our recruitment of new colleagues. Fifty-five of the 111 submitted outputs involve international collaborations, with co-authors in 24 other countries; 75 outputs have co-authors outside Cardiff, spread across 95 institutions (Table 2).

Central to our strategy and culture lies strong support for individuals in pursuit of excellence in their personally chosen directions (Sections 2.1.2, 2.2). Consistent with this are the awards of several fellowships to staff during the review period. Ben-Artzi (EPSRC Early Career Fellow) came to Cardiff in 2017. In 2018, Zhang won a Marie Skłodowska-Curie Fellowship, ranked in first place at the prioritisation panel. Behrend (2019 AMS Robbins prize-winner) held a Schrödinger Institute Fellowship in 2017. Wood secured a Humboldt Fellowship in March 2020. Cherednichenko moved to the University of Bath just after starting his EPSRC Early Career Fellowship; the post he vacated was filled by Eswarathasan, who secured a Clay-LMS grant in his first year in post.

Institution	No. of outputs
University of Oxford	6
Imperial College London	5
University of Bath	4
University of Mannheim	4
University of Padova	4
Purdue University	3
TU Munich	3
University of Alberta	3
University of Kent	3
University of Münster	3

Nation	No. of outputs
Germany	22
Italy	10
Canada	8
Australia	7
France	6
Austria	3
Belgium	3
Switzerland	3
China	2
Croatia	2

Table 2: Top 10 collaborating institutions and nations by submitted outputs

1.3 Interdisciplinarity

Supported by fundamental research from across the School, our Applied and Computational Mathematics, OR, and Statistics groups particularly thrive though interdisciplinary interaction. Interdisciplinarity is sustained long-term by our recruitment strategy (Section 2.1.1) which supported collaborations with Computer Science and Biosciences (Gagarin, Kaouri, Woolley) and the ONS (Mitra). As evidenced throughout the statement, our collaborators include Computer Science and Informatics (via Gagarin, Harper, Lewis), with whom we shall soon be co-located (Section 3.3); Cardiff University Brain Research Imaging Centre (Balinsky); Cardiff Business School (Chen, Liu); the ONS (Gillard, Mitra); Engineering (Kadri, Mihai, Pepelyshev, Phillips); and the Cardiff University Water Research Institute (Jones, Strokorb). These collaborations were supported by external research funding (Sections 3.2, 4.3.2), and by University investment (REF5a). Our commitment to interdisciplinarity is further seen in delivery of our workshops and conferences (Section 4.4) and in our SIAM-IMA Student Chapter, one of the first SIAM Student Chapters in the UK.

1.4 Impact and engagement

Cardiff's reputation for working successfully with industry has been built up over forty years, with an evolving strategy based on (i) engagement with users and identification of resulting impact; (ii) an agile approach to opportunities; (iii) supporting staff to generate impact from their research, and recognition and reward of the impact achieved.

(i) Engagement with users and maximisation of impact are achieved by close collaboration with users and beneficiaries from the outset. Senior employees from our industrial partners serve on our Industrial Advisory Board, including representatives of Admiral, BA, BT, Dyson, Ernst & Young, Lloyds, McLaren, Nationwide, ONS, Tata Steel, Welsh Government and Welsh Water, and help to guide our impact strategy and identify collaborative opportunities. The value placed in our successful collaboration with external partners is demonstrated by the posts they have funded in Mathematics. Eighteen years of postdoctoral research in the REF period were financed through income from industry and the NHS. Table 5 in Section 4.2 illustrates the breadth of funding partners. The lectureship of INFORMS prizewinner Gartner was 100% funded by NHS Wales for the first 44 months and is still 20% funded by them now, on a rolling annual basis.

(ii) Our agile approach to opportunities saw us establish a new collaborative relationship with Crimtan, a global digital marketing technology and trading consultancy, and secured a new Senior Lectureship in Data Science, created as part of a strategic partnership between the ONS, the Cardiff Data Science Academy, and the Cardiff Data Innovation Research Institute, of which Harper is joint director. The appointments of Jones to a Chair in OR and to the Board of Cardiff University Water Research Institute, and other new staff into the Unit, including Chen and Liu in mathematical finance (Table 1), substantially enhanced our impact-facing research themes, diversifying our impact strategy.

The development of themes aligned to Computer Science was also strategically important for impact generation. The School now has three Fellows of the Software Sustainability Institute: Knight (impact case study author), Glynatsi and Palmer (2019 OR Society Doctoral Award winner). Working with Cardiff computer scientists Corcoran and Spasić, and with psychologist Krzemiński in the Cardiff University Brain Research Imaging Centre, allowed Balinsky (Mathematical Analysis Group) to develop a machine learning approach to language processing for medical records, significantly outperforming the best existing methods in identification of heart conditions. Balinsky's work with HP Labs also relies on machine learning, built on his expertise in operator theory.

(iii) There are several ways that we enable and reward impact generation. Since 2009 we have had, as Director of Impact and Engagement, Harper, recently elected Companion of the OR Society and winner of numerous other distinctions (see Section 4.1). Harper is supported by Thomas, research manager for the College of Physical Sciences and Engineering, and the University impact team (REF5a). In the School our full-time Knowledge Exchange Officer, Emery, has expertise and contacts that have been vital for developing and organising impact workshops and events (see Sections 4.2, 4.3 for examples). Clear strategic management ensures that impact generation is supported, and colleagues are recognised, for instance through promotion (Section 2.1.2) or nomination for Cardiff University `Celebrating Excellence' or 'Innovation and Impact'

awards. Our monthly School newsletter usually features articles highlighting successful School engagement and impact activities.

Our workload allocation model ensures, as mandated by the University, that colleagues involved in innovation and engagement are given time for these valued activities. Colleagues are encouraged to use the University's Research Leave Fellowships both for fundamental inquiry and for engagement and innovation (see REF5a). As part of our strategy for partnership with ONS, Gillard was seconded there under our EPSRC Impact Acceleration Account, to work on problems in anomaly detection, for one day per week over six months.

A final example of our impact strategy lies in our postgraduate programmes, including a new MSc in Data Analytics for Government developed with ONS. In fact, all our MSc degrees are linked to collaborations with external stakeholders, where students undertake placements and in which many of our MSc and PhD students ultimately accept positions. Over the REF period, 70 of our MSc students secured permanent positions with our Industrial Advisory Board partners alone. Among our former PhD students, Vile, 2014 SET for Britain prizewinner, is now Senior Implementation Facilitator at the National Institute for Health and Care Excellence; Dadachev works for Google in Zurich; and N. Mohammed is employed at HP Labs. Table 3 in Section 2.3, giving career destinations of PDRAs, contains further examples. These human relationships cement the trust we enjoy with our partners, ensuring long-term vitality of our impact activities.

1.5 Doctoral training

Following a new DTP Agreement between Cardiff University and EPSRC, in 2017 we implemented a system whereby all potential PhD supervisors submit project descriptions to a selection panel organised in partnership with the College of Physical Sciences and Engineering, which ranks them and advertises the best. This system ensures that good interdisciplinary projects are also able to access funding. Since 2017 we obtained approximately five DTP-funded students per year, comparable to the very best we ever had under direct allocation from EPSRC to the School. A small number of studentships are reserved for new supervisors. The School shortlists candidates for interview, ranking by academic merit. During 2012-16, 26% of our DTP-funded intake had first degrees from outside Cardiff; since the adoption of the new system in 2017, this has increased to 61%. We recently negotiated memoranda of understanding with Beijing Normal University, Beijing University of Technology and Xiamen University, including agreements on the provision of joint supervision of doctoral students, involving at least one year in Cardiff. For further details of doctoral training and evidence of success, see Sections 1.8, 2.2.

1.6 Research integrity and research evaluation

All academics and PGR students receive research integrity training, as described in REF5a. Research ethics fall under the School's Research and Innovation Committee. The Director of Research is responsible for monitoring compliance, and for the School's engagement with development of ethics policies at University level. Draft University-level documents are discussed at the School Research and Innovation Committee and feedback provided.

In practice, few of our projects require ethics approvals. Those affected typically involve the College of Biological and Life Sciences (BLS). Such proposals are first approved by the relevant BLS ethics committees before passing to the School for review.

Consistent with Cardiff University's adoption of the San Francisco DORA agreement, the School has continued to evaluate work on scientific merit rather than by metrics or perceived journal prestige. This applies when selecting publications for REF, when recruiting and mentoring staff, and when evaluating cases for promotion. We value software as a research output and for its impact (see Knight's Python impact case study). The School followed the institutional REF Code of Practice at every stage, including output and impact assessment, and determining independence of researchers. Outputs were considered by two independent reviewers and a moderation panel, with additional reviews in cases of disagreement.

1.7 Open Access (OA)

The School supports OA compliance with an administrative team who deposit all accepted papers in the institutional repository, along with software and data. Reminder emails are sent with

sufficient frequency to ensure that deposit deadlines are met. New staff are informed of OA requirements at induction. Our OA compliance rate is over 95% for all works during the period, not only those submitted for review. Our web content management system automatically updates staff pages with links to outputs in the institutional repository. The importance of accessible research is emphasized at the School's funding workshops (Section 3.1) and over 90% of colleagues further enhance access to their research, including preprints and software, using global platforms. Brown has been an **arXiv** moderator for twenty years.

1.8 Covid-19 Response

Members of our OR, Statistics, and Applied and Computational Mathematics groups, including Harper, Kaouri, Wooley and Zhigljavsky, have been extensively consulted by the Welsh Government and its Technical Advisory Group during the pandemic. They were key to facilitating a safe campus and optimising the design and deployment of our in-house asymptomatic testing service. They also participate in the UK-wide OR response through the Royal Society's RAMP initiative. Three of our doctoral students (Henley, Moore and Ostler) won a £3,000 first prize in the NERC Covid-19 Digital Sprint Hackathon 2 for their work on environmental sustainability of public transport under social distancing. Kaouri received research funding from the Welsh Government for her work on anti-viral coatings (Section 4.2.1); she is now a member of the Technical Advisory Group Environmental Sciences subgroup.

The School quickly moved its research seminars online, allowing these vital activities to continue and, indeed, double or triple their audiences. As part of the institutional response (see REF5a), we surveyed researchers and identified two PhD students and one academic whose home circumstances necessitated an early return to office working, as well as others for whom we purchased vital remote working equipment. In the medium term we are working to address the challenges faced by researchers wishing to work abroad, such as during study leave.

1.9 Future Research and Impact Strategy

Our plans for the future are reviewed regularly by our Research and Innovation Committee through formal consultations with colleagues and in light of national and global developments. Excellence, equality, impact and internationalism are key elements in our plans for growth. In particular:

- We will continue to develop a new generation of research leaders through recruitment and career progression of outstanding scholars, and aim to convert the Athena SWAN Bronze Award which we have held since 2015 to a Silver Award at least.
- Just as we have expanded OR beyond healthcare, we shall now enhance impact opportunities by expanding applied and computational mathematics in user-facing directions, building on our investments in mathematical biology, and ensuring that we are well placed to take advantage of the Bond Review.
- In analysis and in probability we now have more researchers using geometric methods, raising exciting prospects for intradisciplinary collaborations with members of the GAPT Group. We aim to expand GAPT by investing in geometry, and in areas of topology and discrete mathematics with applications to problems of shared interest with Computer Science and Informatics.
- Increasing intradisciplinary and interdisciplinary collaborations are already building up critical mass, enabling access to new opportunities for postdoctoral and postgraduate funding. The University's approach to ESPRC-DTP studentship funding has opened up new opportunities for cross-disciplinary interactions, as seen through co-supervision with Computer Science and Informatics.

Most importantly, we shall continue to ensure that our future plans are developed in a democratic and inclusive environment in which all colleagues are encouraged to participate fully in decisions about strategy and direction.

2. People

2.1 Staffing strategy and staff development

2.1.1 Staffing and recruitment strategy

A key element of the expansion strategy described in Section 1 has been to review our research capabilities regularly and draw up priorities for recruitment. These are based on assessments of the strengths of the research groups by Heads of Group and the School Board, and the identification of opportunities for inter- and intradisciplinary collaboration, as presented in Section 1.4. Our strategy also takes account of national priorities articulated by UKRI, EPSRC's Community Overview Documents, and strategic priorities of the University, alongside our fundamental commitment to equality, diversity and inclusion (Section 2.4). The identification of cross-cutting themes, which enable us to expand our research base while maintaining both excellence and coherence, benefitted immensely from the advice of our External Advisory Panel, whose members are currently Martin Bridson FRS (Oxford), Michael Grove (Birmingham), Boris Khoruzhenko (QML), Robert Leese (Oxford), Sally McClean (Ulster), Alastair Spence (Bath) and Henry Wynn (LSE).

The appointments made in each group have always met more than one strategic objective. For instance, Jones expanded OR in environmental applications, establishing a new collaboration with Earth and Environmental Sciences. Ekström (Earth and Environmental Sciences) and Strokorb, a new probabilist shared between Analysis and Statistics, worked with Jones to win the 2018 RSS Mardia Prize. Their work links three of our five groups in an interdisciplinary collaboration between scientists who had not met before coming to Cardiff. Similarly, the appointment of applied combinatorialist Andrei Gagarin (OR Group), an initiative to support collaboration with Corcoran and Spasić (Computer Science), complements the theoretical work of our AMS prize winner Behrend (GAPT Group). Corcoran and Gagarin are joint leaders of the Discrete Mathematics and Data Science Research Team, a 'research semigroup' composed of researchers from both schools. Kaouri and Woolley (Applied and Computational Mathematics) expanded our collaborations with life sciences and organised major meetings such as the European Study Groups with Industry.

Ensuring that each appointment meets multiple objectives enabled us to make strong cases for additional University investment. The pure mathematics posts of Lechner, Pennig and Wood are all the direct result of University investment in our critical mass, as are the posts of Chen (mathematical finance $\subset$ OR), Gauthier (statistics), Strokorb (analysis, statistics) and Liu (mathematical finance $\subset$ OR). Cardiff University also supported the School by ensuring that vacancies are filled quickly, an important mechanism for ensuring optimistic staff morale and an upbeat environment.

2.1.2 Staff development and promotion

Most Teaching and Research staff recruited since 2014 commenced on contracts with three-year probationary periods. The School and the University provide several support mechanisms, some tailored to probationers and some available to all. In particular:

1. Probationers receive a substantially increased allowance for research in their workload model, to enhance development of their longer-term research career and the formulation of research proposals. The Director of Research proactively collaborates with probationers, with ECRs and with Heads of Group to identify funders, providing successful past applications, drawing in colleagues with appropriate expertise, and giving practical advice on processes. The MSCA Fellowships of Zhang (Section 4.1) and Rösler, and the Humboldt Fellowship of Wood, are three examples of this system in action.

2. The University has an intensive bespoke programme of support for Fellowship applicants (e.g. UKRI, ERC and Marie Skłodowska-Curie): in particular, experienced scientific colleagues from across the College help applicants to create accessible, engaging proposals, support is provided for responses to reviewers, and with mock interviews.

3. The School encourages staff to join the EPSRC Peer Review College and Associate Peer Review College. Ten colleagues are College members; D.E. Evans sits on the EPSRC Strategic

Advisory Team. Engagement with other funders is also important, with Leverhulme Trust and London Mathematical Society being particularly valuable partners in fundamental research.

4. New staff receive an enhanced personal development fund, typically totalling £7,000. Through careful mentoring, they are encouraged to use these funds ambitiously. Many of our distinguished visitors (Table 4 in Section 4.1) were funded in this way, or through the University's International Collaboration Seedcorn Fund (e.g. ICM speaker Masaki Izumi, AMS Fellow Murad Taqqu).

5. All staff may request Study Leave after five years' service; they may also apply to the University Research Leave Fellowship Scheme for travel funds and to secure replacement teaching costs for the School (REF5a). The success of the scheme is seen in research outputs and three of the highlighted projects in Section 3.2 were developed during Study Leave (PlASMA, IVP-MAG, LADS).

6. We have a 100% completion rate for staff Performance Development Reviews since this new system's introduction in 2016. One indicator of its effectiveness is the success rate of colleagues applying for promotion. During 2018-19 and 2019-20, all 19 colleagues who applied were promoted (73% of eligible staff; 83% of female staff). The University's Academic Promotions Committee is independent of the School. Personal chairs for research excellence were awarded to Behrend, Chen, Dirr and Schmidt during the REF period.

2.2 Training and support of ECRs and doctoral students

The quality of our doctoral training is reflected in the career destinations of our PhD graduates: 15 out of 46 graduates continued to academic positions in institutions including Bath, Ben-Gurion University, Bologna, Bristol, Lancaster, Max Planck Plön and Warwick, as well as Cardiff; industrial employers include Fondazione Bruno Kessler, Google, HP Labs, Renishaw, the NHS and NICE. Contacts made in our strategic research workshops (Section 4.4) and impact events (Section 4.2) also enhance career prospects.

Equipping doctoral students with appropriate technical skills is vital to their success in research; consequently, the School has provided taught courses for the MAGIC Consortium ever since joining, principally in Functional Analysis, Spectral Theory and Numerical Solution of PDEs. With the other mathematics departments in Wales, we direct an annual meeting at Gregynog Hall, involving three eminent external speakers and parallel sessions in which PhD students are encouraged to present their work. All our research students attend their group seminars and produce annual written reports. A supportive annual viva voce assessment helps them target their efforts on work whose importance they can fully articulate. Consequently, our students have won prizes for their work and exposition: the 2017 Mike Crisfield prize of the UK Association for Computational Mechanics for Safar, the best poster award for Aspland at OR60 (2018) and for Williams at ORAHS (2019), and the OR society 2018 doctoral award for Palmer. Groves and Morgan also presented their work at STEM for Britain in the Houses of Parliament. Our students run an active SIAM-IMA student chapter.

Supported administratively and financially by the School, our PGR student body, on their own initiative, established an internal seminar series, reading groups, and a special initiative in the Work@1 Programme encouraging undergraduates to consider research careers. They also set up a 'Women in Mathematics' forum, with a dozen regular attendees. Existing PGR students were particularly helpful to us in increasing recruitment of female research students. Accordingly, we greatly enhanced the visibility of our female staff and students in our online and printed course materials. We give individual encouragement to potential applicants and highlight PhD study and the funding available through the 'Women in Mathematics' forum and Work@1 sessions. This resulted in substantial improvements in female representation since 2017/18, at each recruitment stage (Fig. 1), further evidence of our commitment to equality, diversity and inclusion (Section 2.4).

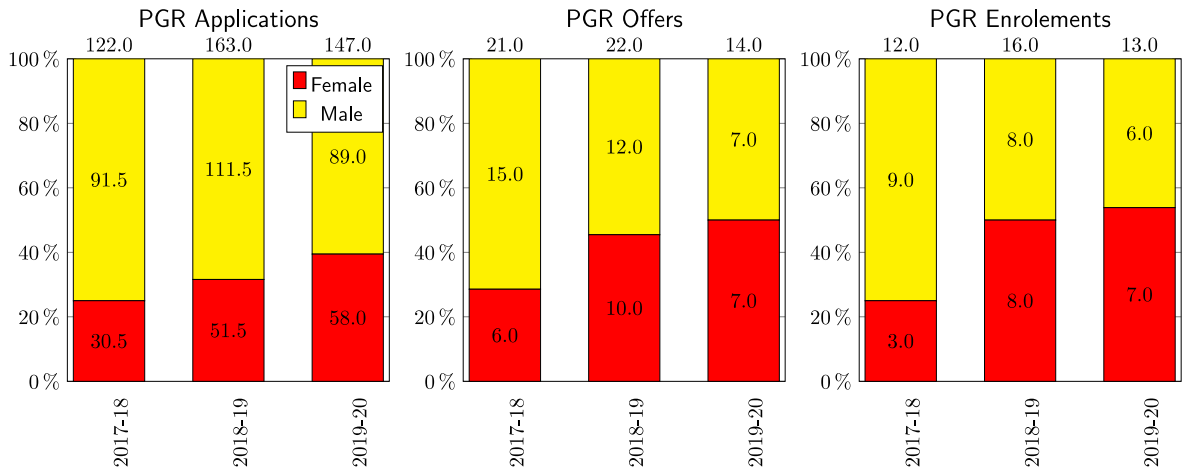

Figure 1: PhD applications by gender

The annual University Postgraduate Research Experience Surveys confirmed overall satisfaction rates averaging 93% for Maths PGRs, with satisfaction rates for research supervision averaging 95%. The increased EPSRC DTP and CASE allocations we received from the University (see Section 1.5), combined with our strategies of collaboration between research groups and with other Schools, encouraged colleagues to produce exciting collaborative PGR projects, such as modelling for optimal electric vehicle recharging locations with Computer Science. By connecting these, where appropriate, to proven non-UKRI funding opportunities (see Section 4.2) and by securing new Horizon Europe Doctoral Training funds, we aim to continue expanding our future PGR funding.

2.3 Career development of postdoctoral research assistants and Fellows

Our former PDRAs and Fellows progressed to a variety of academic and industrial careers (Table 3). PDRAs moving into industry are often employed by our partners, delivering ongoing impact from the research they conducted in the School. For many PDRAs, the most important element of preparation for future careers, beyond a strong portfolio of outputs, lies in research funding. Section 3.1 describes how we support grant applications. Since teaching experience is valued by academic employers, PDRAs also have the opportunity to deliver a small number of lectures, typically not more than one 27-lecture module during a three-year appointment. Our Mathematics Education Group, led by 2019 Advance HE Fellowship winner Wilson, provides maths-specific support for teaching.

PDRA/Fellow	Current Employment
Sabine Bögli	Assistant Professor, Analysis of PDEs, University of Durham
Soheil Davari	Senior Lecturer, University of Hertfordshire Business School
Ermal Feleqi	Head of Mathematics Department, Universiteti i Vlöres, Albania
Penny Holborn	Senior Lecturer in Maths, Stats & OR, University of South Wales
Ahmed Kheiri	Lecturer in Operational Research, Lancaster University
Izabela Komenda	Mathematical Modelling Unit Lead, ABCi, Aneurin Bevan University Health Board
Rishot Minty	Analyst, Department for Work and Pensions, London.
Jennifer Morgan	Performance Analysis Manager, Delivery Unit, NHS Wales
Baptiste Morisse	Mathematical modelling consultant, Sivienn, Paris
Duc Vinh Nguyen	Lecturer-Researcher at Tonducthang University, Vietnam

Juan Reyes	Research Associate, Instituto de Estadistica de las Islas Baleares
Otogo Uuye	Director of the Institute of Mathematics, National University of Mongolia, Ulaanbaatar
Julie Vile	Implementation Facilitator (Wales), National Institute for Health & Care Excellence
Cheryl Voake-Jones	Skills Development Manager at University of Bath
Hayley Wyatt	PDRA, Cardiff School of Engineering
Pre-2014 PDRA/Fellow	**Permanent position secured post-2014**
Shane Cooper	Lecturer in Applied Mathematics, University College London
Paul de Medeiros	Associate Professor of Applied Mathematics, University of Stavanger

Table 3: Current employment of past PDRAs/Fellows in Cardiff School of Mathematics

2.4 Promotion of equality, diversity and inclusion (EDI)

Promotion of EDI touches on every aspect of the School's life and is central to our strategy of ensuring all talent is supported by organisational processes and culture. We aim to achieve at least a 1/3 female shortlist for every academic position. Job adverts contain contact details for both a female and a male colleague. Since 2014, numbers of female teaching and research staff increased significantly, from one to seven. Every appointment panel has at least one female member. All panel members receive enhanced EDI and Unconscious Bias training.

The School established procedures to support colleagues returning after periods of absence, ensured collegial timings of seminars and meetings (10:00-16:00), and provided enhanced help for those with health and wellbeing needs. Several of the papers submitted to REF were written by researchers we assisted through periods of difficulty, whose successful return is also demonstrated by promotion.

Regardless of gender, colleagues are encouraged to take parental leave entitlements. The School allows teaching-free periods for those returning from maternity leave. We promoted our Returner's Scheme, developed in 2014, to the University; it is now adopted across the institution. Staff meet the School's HR Officer for details of University support available before going on leave, and additional School support available on their return, including protected study time and flexible working. Our Head of School chairs the College of Physical Sciences and Engineering EDI Committee, reflecting best practices and School achievements.

The School has a request procedure for staff requiring teaching adjustments due to caring responsibilities, disability, religion etc. This allows planning and promotion of flexible working. The numbers of teaching timetable adjustment applications each year since 2015 were 6, 14, 10, 9 and 12, respectively. One of the requests each year was from a female academic; the rest were male. We adjust timetables at either the beginning or end of the day for all applicants. Colleagues requesting adjustments at both ends of the day apply using the University's flexible working policies. Since 2015, two such requests were received from academics; both were approved.

We monitor the gender balance of all seminar series. There has been significant change since 2014, with equal numbers of female and male speakers in 2016/17 for the most high-profile series, the School Colloquium. There are differences between the other seminar series' gender balance, but most achieve at least 25% female speakers, reflecting the proportion of female researchers in UK mathematical sciences.

Progression from our Athena SWAN bronze award to silver, and subsequently gold, is an important future goal. The increase in the number of female researchers since our previous application, promotions of female academics to senior positions, and our support of high-visibility activities by female researchers (see Section 4.2) make us optimistic of successful outcomes.

2.5 International staffing

Approximately 2/3 of academics in the School are dual nationals or hold non-UK citizenship only. Aware of the uncertainties facing colleagues who do not hold UK citizenship following the 2016 BREXIT referendum, the University established an International Staff Network and mechanisms to support European colleagues seeking settled status.

The School was aware that international staff were under-represented in its management structure and took steps to ensure that there are truly no barriers to contributing to the School's leadership. In particular, many key strategic roles are now held by international colleagues who came to Cardiff within the last decade. These include Aliev, Director of Postgraduate Studies; Artemiou, Deputy Director of the Data Science Academy; Chen, Director of International Partnerships; Lechner, Head of the GAPT Research Group; Mihai, Deputy Director of Research. All colleagues in these roles were promoted at least once during the assessment period, recognising research and impact successes the School supported them to achieve. Such initiatives also emphasize our commitment to equality and diversity in ethnicity.

3. Income, infrastructure and facilities

3.1 Research funding and strategies for generating income

The School was commended in REF2014 for its strong research income. In the current census period, we surpassed the previous level of grants awarded, with £6,248,250 compared to awards of £6,098,983 during the previous REF census period. We saw large increases in EU grant awards (up from £769,813 to £1,434,942) and Welsh Government awards (up from £37,658 to £388,482). PDRAs and postdoctoral Fellowships also increased from 31 person-years in the previous census period to 40 person-years in the current period.

Though performance was initially driven by established colleagues, from 2018 there was significant growth in awards to new appointees and ECRs, who also benefit from in-School and University mentoring (e.g., for our MSCA Fellowships, Section 1.2). The funding awarded in the two-year period July 2018 to July 2020, £4m, is almost 2/3 of the funding awarded during the whole REF period, in part reflecting the momentum from sustaining new staff recruitment.

Funding from UKRI, the EU, the Leverhulme Trust and the London Mathematical Society is important to the School (see the projects noted in Section 3.2). We have also, however, increased revenue from impact activities. Although publicly available HESA data only provides staff numbers to within five FTE, these data indicate that since REF2014 our per-capita income from UK central government bodies/local authorities, health and hospital authorities has been in the top UK quartile, reflecting our long-term partnerships with the NHS, the ONS, and the Welsh Government. Of the 40 person-years of postdoctoral time secured during the review period, 18 years were funded by industrial partners (Table 5 in Section 4.2) including over 150 months of PDRA time for a Healthcare Modelling Unit in the School. As we grow our research funding, we shall maintain this balanced portfolio of sources (see REF4a/b data), in line with our strategic commitment to societal benefit.

Our strategy for income generation is based on (i) mentoring; (ii) fair workload allocation and respect for EDI; (iii) internal peer review; (iv) funding workshops; and (v) impact events. We already mentioned in Section 2.1 our mentoring systems for supporting new colleagues to generate income. We describe the remaining mechanisms below.

Securing research funding depends on having the time to conduct outstanding research, so we particularly ensure that the workload associated with activities which support EDI, such as Athena SWAN applications, is fairly shared. Given our arrangements for special timetable requests described in Section 2.4, colleagues with family commitments are able to engage fully in research and in income generation. All colleagues are given the support they need regardless of gender, ethnicity, disability or other protected characteristics. We celebrate the 100% success rate of our female applicants in securing both First Grants/NIAs and second grants. The LADS project (Section 3.2) arose from one such First Grant.

All proposals, from researchers at any career stage, pass through a supportive **internal peer review**. Reviewers may submit reviews anonymously, through the Director of Research. We aim

for two reviewers on each proposal and often send proposals to ECRs for review, particularly where they have valuable technical expertise or could learn, from the experience, what a good proposal looks like. Our School Research Manager prepares financial information, checks conformity with funder requirements and assists the Director of Research much as a Managing Editor would assist the Editor-in-Chief of an academic journal.

Since 2016 the School has organised afternoon funding workshops at least annually, covering different funders and application procedures. The topics discussed are decided by popular demand. The Leverhulme awards to Behrend and Cherdantsev, both funding three-year PDRAs, are recent examples which arose from a single grants workshop. We started to see a significant increase in awards generally, approximately 18 months after the first workshop. Both the School, and the University's Research and Innovation Service (RIS), regularly invite funders to visit and explain their strategic priorities. RIS colleagues have been seconded to UKRI for extended periods, gaining in-depth knowledge of their working methods.

One outcome of our funding workshops, aligned to our global visibility strategy, was a decision to propose and direct more research programmes at international institutes. These included programmes at the Isaac Newton Institute, the ICMS in Edinburgh, MF Oberwolfach, Institut Mittag Leffler in Stockholm, RIMS in Kyoto, the Simons Center in New York, and two LMS-Durham Symposia (see Section 4.4).

Our impact events bring together researchers and practitioners. Industrial collaborative partners include Admiral, BA, Crimtan, Ernst & Young, GE, Glaxo Smith Kline, Google, HP, NHS, Lloyds, ONS, PA Consulting, Procter and Gamble, Roche, Tata Steel and the Welsh Government. We secured long-term external sponsorship of open-ended academic posts, including Gartner's lectureship (NHS Wales) and a new Senior Lecturer in Data Science, helped by a strategic partnership agreement between ONS and Cardiff University (with Harper as academic chair). Dyfed-Powys Police part-funded two CASE PhD studentships on crime prediction. Two KESS2/European Social Fund PhD studentships were part funded by Cellesce (a breast cancer organoid research company, led through Cardiff Biosciences) and the London Women's Clinic; four further KESS2 studentships were part-funded by partners related to our impact case studies (Table 5 in Section 4.2).

These developments show that Cardiff is already establishing many of the positive knowledge exchange scenarios envisaged in Sir Philip Bond's `Era of Mathematics' report for UKRI. In fact, Harper was a member of the Expert Review Committee contributing to this report, which has persuaded UKRI to substantially increase mathematics funding.

3.2 Highlighted projects

Research grant funding since REF2014 led to significant research outcomes that are enriching our discipline and the School's research environment. Here we present some examples with lasting impact on our future research directions.

QUEST (Quantitative Estimates in Spectral Theory) is Ben-Artzi's EPSRC Early Career Fellowship project. It resulted in the appointment of four new members of the Mathematical Analysis Group: Ben-Artzi himself, two PDRAs (Morisse and Rösler) and, following the award of a Marie Skłodowska-Curie Fellowship, Zhang. With the skills acquired by working on this project, Rösler secured an MSCA Fellowship on computational complexity in quantum mechanics, which he holds in the School. QUEST funded key meetings in the School, attracted several of the eminent visitors in Table 4 (Section 4.1) and was a key ingredient in the establishment of the South Wales Analysis and Probability Seminar, and the UK-wide LMS Network on Challenges in Non-Selfadjoint Spectral Theory.

PIASMa (Leverhulme Trust RPG-2019-083, PI: Behrend, 2019–2022) investigates the enigmatic connection between plane partitions and alternating sign matrices. It has been known for several decades that these two classes of discrete mathematical objects share remarkable enumerative properties, but the link can only currently be confirmed indirectly. This fundamental problem is being addressed by developing new techniques involving partially ordered sets and lattice paths, which are expected to lead to a substantially enhanced understanding of the underlying combinatorics. Saikia, a PIASMa PDRA, is an excellent example of Cardiff's international

connectivity. He completed his PhD at Universität Wien with Prof. Ilse Fischer, having been awarded a Postgraduate Diploma from Abdus Salam ICTP Trieste and a master's degree from Tezpur University, India.

IVP-MAG (EP/K024078/1, PI: Brown, 2013-2015), a multidisciplinary collaboration between Mathematics and Computer Science in Cardiff and Engineering in Swansea University (Swansea PI: Ledger), studied inverse problems for MIT. This project emerged from the 2011 Isaac Newton Institute programme on Inverse Problems, chaired by Brown, and was succeeded by MIVP (EP/P005985/1, PI: Brown), a network encompassing nodes in Bath, Bristol, Cambridge, Cardiff, Edinburgh, Kent, Leeds, Manchester, Nottingham, Southampton, Swansea and UCL. IVP-MAG developed new Maxwell solvers for Free-FEM++, as well as fundamental publications on solvability of inverse problems with boundary measurements on arbitrarily small open sets. Two postdocs secured research careers in industry, one at strategic partner Renishaw; a third secured a permanent lectureship in Computer Science in Cardiff.

MEMSI (Modelling Emergency Medical Services in Indonesia), an EPSRC GCRF funded project (EP/T003197/1, PI: Harper) develops spatio-temporal forecasts of demand, with stochastic optimisation and simulation decision support tools for emergency and disaster relief planning across Indonesia. It builds both on novel underpinning research to uniquely capture the behaviour of queueing systems with multiple priority classes and time-dependent arrivals (EP/F033214/1), and on the development of an open-source discrete-event simulation shell (funded by NHS Wales). This research created additional impact for the Welsh Ambulance Service by informing them of optimal staffing levels and associated rosters.

LADS (Limit analysis of debonding states in multi-body systems of stochastic hyperelastic material, EP/M011992/1, 2015-2017; PI: Mihai) combined knowledge from elasticity, statistics and probability to develop sophisticated tools for modelling elastic material behaviour in diverse areas of science, medical research and manufacturing. The project PDRA, Wyatt, now holds a University lectureship in Engineering. The importance of this research was recognised by the award of a second EPSRC grant (EP/S028870/1) allowing translational research to test the mathematical results on a wide range of materials of practical interest. As important challenges are associated with quantification of uncertainties in material responses, the long-term goal is to develop stochastic elasticity methods as powerful tools for future applications.

3.3 Infrastructure

We currently occupy a 3500m^2 building, with a dedicated Mathematical Sciences Library, numerous seminar and collaboration rooms, and lecture theatres accommodating between 30-190 people. In 2016 a business case was made with the School of Computer Science and Informatics for co-location in a new building, based upon the identification of twenty researchers with aligned interests and upon the strategic decision (see Section 1) to expand joint research in areas including discrete mathematics, combinatorics, optimisation and applied probability, which will also enhance collaboration with the ONS. The new £39m 9,700m^2 facility, named Abacws, was designed by Stride Treglown and Adjaye Associates, collaborating closely with academics from both Schools. Currently nearing completion, its location is in immediate proximity of Cardiff University's Brain Research Imaging Centre (CUBRIC), a £44m flagship centre with whom Balinsky and Spasić already collaborate.

Our co-location with Computer Science and Informatics will greatly benefit the University's Data Innovation Research Institute, that is jointly led by the two Schools. New hot-desking will support researcher engagement across the University, directly benefitting our Unit. The new space will be essential as we emerge from Covid-19 in supporting new interactions in Mathematics and in growing our collaborations with Computer Science and Informatics (Gagarin, Harper, Lewis, Balinsky), particularly at the interface with combinatorics, OR and machine learning. In the long term, the Abacws Building will accommodate growing numbers of PDRAs and allow strategic expansion of our international visitor programme, conferences, and impact events, all key components of our environment (Section 4). The long-term presence of eminent visiting scientists, who will have the opportunity to visit two dynamic Schools in one building, will be particularly

important for the personal development of our ECRs, whose scientific prospects will be further enhanced in an intrinsically multidisciplinary environment (see Section 1.3).

The School already shares support for research and finance with Computer Science and Informatics. This will extend to IT in the new building. For HPC we currently use the hardware and support of Advanced Research Computing at ARCCA (REF5a), supported by the University and European Structural funds via the Supercomputing Wales project, to conduct CPU-intensive simulations in OR, for data-intensive work in collaboration with CUBRIC, and for CFD to support our research on non-Newtonian flows.

4. Collaboration and contribution to the research base, economy and society

4.1 Cardiff and the global research base

4.1.1 Distinctions and Prizes

The School celebrates the successes of colleagues whose contributions to the research base and society have been recognised with distinctions and prizes.

Roger Behrend, a Cardiff mathematician for twenty years, won, with co-authors Ilse Fischer and Matjaž Konvalinka, the 2019 American Mathematical Society Robbins Prize for the paper *Diagonally and antidiagonally symmetric alternating sign matrices of odd order* (Advances in Mathematics 315, 324-365 (2017)) in which they prove, after more than thirty years, a formula for the number of odd-order diagonally and antidiagonally symmetric alternating sign matrices. This was the last open conjecture of David Robbins on alternating sign matrices. Behrend has now been awarded a Leverhulme grant funding a postdoctoral position for Saikia, to investigate topics arising directly from the prize-winning work. Behrend was widely recognised before the award: since 2014, he gave seven guest lectures on his research in Europe and Japan, including the two talks in Table 6.

Paul Harper became, in 2019, the youngest ever recipient of the Companion of Operational Research Award, for sustained support for the development of OR and outstanding service to The OR Society and the wider OR community. This prize is the most prestigious in a series of distinctions which include the 2015 Times Higher Education Award for Outstanding Contribution to Innovation, two Cardiff University prizes for Innovation and Impact (2011 and 2015) recognising Harper as one of the University's top researchers for the societal benefit of his work, and the 2017 Cardiff University `Celebrating Excellence' award for Innovation and Enterprise. Harper's research interests concern stochastic OR, including queueing theory, simulation methods, optimisation, and game theory; mathematical modelling of healthcare systems; modelling for the prevention, detection and treatment of diseases; machine learning and data mining. The author of more than eighty peer-reviewed papers, he is a named investigator on over £11m of grants.

Owen Jones and **Kirstin Strokorb**, with co-author **Marie Ekström** (School of Earth and Environmental Sciences), won the 2018 Royal Statistical Society Mardia Workshop Prize to support their interdisciplinary work on extreme weather. Jones is an applied mathematician with a background in data analytics, optimisation and simulation who has collaborated with the Australian Department of Agriculture, Fisheries and Forestry, McLaren F1, and the UK's National Air Traffic Systems. Strokorb is a probabilist focusing on stochastic processes and dependence concepts in extreme value theory, a branch of probability and statistics that provides theoretically sound procedures for extrapolation beyond the range of data, including proper understanding of the limitations of such procedures. Following her doctorate from Göttingen in 2013 and a three-year assistantship in Mannheim, Strokorb was appointed to a University lectureship in 2017. She delivered more than twenty invited presentations in Europe and North America since 2015.

Junyong Zhang was awarded a Marie Skłodowska-Curie Fellowship with a 100% score and ranked top of all the mathematics applicants in 2018. He joined us from the Beijing Institute of Technology, where he maintains his affiliation. Zhang's interests include harmonic analysis, spectral analysis and PDEs. He studies problems related to the long-time behaviour of nonlinear dispersive equations, as well as Strichartz and restriction estimates, particularly for problems on nontrivial underlying manifolds. He obtained his PhD in 2011 at the Institute of Applied Physics

and Computational Mathematics in Beijing and has since spent a year at both the Australian National University and Stanford University.

Anatoly Zhigljavsky won the 2019 Constantin Carathéodory Prize for his lifetime contributions to global optimisation. Zhigljavsky obtained his Doctor of Science degree from Leningrad in 1987 and came to the University twenty years ago as Professor of Statistics. He is the Head of the Statistics Group and was Director of the Centre for Optimisation and its Applications. His research interests are time series analysis, multivariate statistical analysis, statistical modelling in market research, and stochastic global optimisation. The author of more than 100 research articles, he has long-term industrial collaborations with Procter and Gamble, GlaxoSmithKline and Crimtan.

4.1.2 Outstanding visitors and collaborators

We host many eminent visitors, either as part of research projects in the School or as speakers in our colloquia and conferences. Table 4 shows some of the distinguished scholars hosted since 2014, including five Fields Medallists and several ICM invited speakers. Many are involved in fundamental blue-skies research with members of the School, which is one of the strategic drivers for inviting them. **Jesus de Loera**, **Martin Henk**, and **Robert Weismantel** work with Aliev and Oertel; **Valery Smyshlyaev** works with Cherdantsev; Pennig made reciprocal visits to **Constantin Teleman** and has joint work with **Yasuyuki Kamahigashi**; he has recently started a new research project with **Masaki Izumi**; Dirr is a collaborator of **Panagiotis Souganidis**; Ben-Artzi is working with **Anders Hansen**; Leonenko works with **Murad Taqqu**, and number theorist Lettington is researching Bohemian matrices with **Nick Higham**. Mihai was invited speaker at the 70th birthday meeting for **Sir John Ball** in Oxford. **Maciej Zworski** visited to collaboration with Eswarathasan.

Fields Medallists	Michael Atiyah FRS, Alain Connes, Simon Donaldson FRS, Vaughan Jones FRS, Martin Hairer FRS
Prize winners	John Ball FRS (von Karman prize), Jonathan Bennet (LMS Whitehead prize), Mike Carter (four Canadian OR Society prizes), Alessio Corti (Junior Whitehead prize), Allison Etheridge FRS (Senior Anne Bennet prize), Ivar Ekeland (D'Alembert prize), Isabelle Gallagher (Doistau-Blutet prize), Anders Hansen (Whitehead prize), Martin Henk (von Prechtl medal, Hlawka prize), Nicholas Higham FRS (Whitehead prize, Fröhlich prize), Jon Keating FRS (Fröhlich prize), Frances Kirwan FRS (Senior Whitehead prize), Heinz Langer (Nagy medal, Kreĭn prize), Jens Marklof (Philip Leverhulme prize, Whitehead prize), James Maynard (SASTRA Ramanujan prize, Whitehead prize), Clément Mouhot (Whitehead prize, Adams prize, Grand Prix Madame Victor Noury), Barbara Niethammer (Richard von Mises prize, Whitehead prize), Felix Otto (Leibniz prize), Alexander Pushnitski (Whitehead prize), Miles Reid FRS (Berwick prize, LMS Pólya prize), Carola-Bibiane Schönlieb (Whitehead prize, Philip Leverhulme Prize), Valery Smyshlyaev (Whitehead prize), Panagiotis Souganidis (College de France medal), Andrew Stuart FRS (Whitehead prize, Dahlquist prize), Nick Trefethen FRS (IMA gold medal, LMS Naylor prize, SIAM Pólya prize), Gunther Uhlmann (Böcher Memorial prize, Kleinmann prize), Stefaan Vaes (Royal Flemish Academy prize 2012, Francqui prize 2015) Dima Vassiliev (Whitehead prize), Andras Vasy (Böcher prize) Karen Vogtman (LMS Pólya prize), Trevor Wooley FRS (Salem prize, Fröhlich prize), Maciej Zworski FRSC (Coxeter-James prize)
ICM Speakers	Isabelle Gallagher, Mark Gross FRS, Colin Guillarmou, Masaki Izumi, Dmitri Kaledin, Yasu Kawahigashi, Frances Kirwan FRS, Clément Mouhot, Barbara Niethammer, Miles Reid FRS, Caroline Series FRS, Andrew Stuart FRS, Constantin Teleman, Nick Trefethen FRS, Gunther Uhlmann, Andras Vasy, Karen Vogtmann, Robert Weismantel, Trevor Wooley FRS, Maciej Zworski FRSC

Fellows of the AMS	John Ball FRS, Andrew Granville, Frances Kirwan FRS, Jesus de Loera, Caroline Series FRS, Murad Taqqu, Stefaan Vaes, Andras Vasy, Karen Vogtmann, Trevor Wooley FRS

Table 4: Distinguished visitors to the School.

4.2 Collaboration and the economy

The economic beneficiaries of our research include commercial/industrial organisations with global reach, such as HP and Google, together with public organisations which benefit from savings through enhanced organisational practices. We helped global companies to optimize their digital advertising (see impact case study with Crimtan) and benefitted educators, researchers and industry through mathematical consultancy under-pinned by fundamental research, as well as a range of open source software.

4.2.1 Mechanisms for engagement with users

As mentioned in Section 1.4, our Innovation and Engagement Committee receives strategic advice from our Industrial Advisory Board comprised of senior personnel from more than 15 organisations. External partners (see also Section 3.1) are further engaged through contributions to research-led teaching. Many offer three-month projects on our MSc programmes, often leading to deeper collaborations. After hosting MSc projects for some years, Public Health Wales (PHW) funded a PDRA during 2018-2020 who worked closely with the Home Office and all police forces and commissioners across Wales to help inform the national approach to policing vulnerability; that is, how to optimally design and operationalise services to support those suffering domestic violence and adverse childhood experiences. PHW is now a significant stakeholder in our modelling work on public health delivery logistics. Airbus recently awarded £120,000 to fund research in collaboration with Engineering and Mathematics in the University on the use of acoustic sensors to monitor airframe damage. Led by Pullin (Engineering) with Gillard as Mathematics co-I, this project was enabled by connections made by Gillard through the Data Innovation Research Institute.

We also engage businesses through partnerships which allow both them and us to access research funding. A topical recent example is Kaouri's collaboration with Smart Separations on antiviral coatings, funded by Business Wales, with £58,000 coming to the School. This is joint research with Oxford University.

4.2.2 Industry-academia funding

Since the last REF we secured in excess of £1.6m directly from industry and charities to fund research amounting to 13 PhD studentships, 18 funded years of PDRA time, and eight years towards a match-funded lectureship: see Table 5. This table does not include the 50% ONS funding of Mitra's Senior Lectureship, which commenced after the 31/07/2020 census date.

Income	Sponsors
Two EPSRC CASE PhD scholarships	ONS; Dyfed-Powys Police
Six KESS2/European Social Fund PhD scholarships	Welsh Government, Welsh Blood Service, Velindre Cancer Centre, Cellesce (breast cancer organoid research), NHS Wales, London Women's Clinic
26-months PDRA	Cancer Research UK
Nine-months PDRA	The Health Foundation
12-months PDRA	PHW / Home Office
17-months PDRA, two PhD scholarships	Cardiff and Vale University Health Board
One PhD scholarship	Cwm Taf University Health Board

One joint lectureship, 153 months PDRA time and two PhD scholarships	Aneurin Bevan University Health Board
Sponsorship of 146th European Study Group with Industry	ExxonMobil, KPMG Cyprus, Smart Separations
£120,000(jointly with Engineering): acoustic airframe monitoring	Airbus UK

Table 5: Industrial income

4.2.3 Networks and industrial study groups

Several networks are of importance to our strategic objective of expanding impact beyond OR and Statistics. The School belongs to the European Consortium for Mathematics in Industry, a pan-European consortium of academic organisations and companies. Kaouri was a core member of MI-NET (EU COST Action, May 2015-April 2019) which involved industrial mathematicians from thirty-two countries. She is a core member of the SciShops.eu project (H2020, Science with and for Society) where organisations in twelve countries work with social representatives to identify and solve pressing societal challenges through collaborative research. Additionally, Harper is a member of the Expert and Advisory Board of SciShops.eu.

In December 2018, Kaouri co-organised the 146th European Study Group with Industry, with the EU-funded Mathematics for Industry Network MI-NET and the EU project SciShops.eu. This industry-academia workshop took place at the University of Cyprus. Thirty-seven ECRs and experienced researchers from ten countries worked on three business and societal challenges, in collaboration with non-academic organisations: optimizing filter performance (Smart Separations - UK start-up company), fuel reduction in tugboat operations (Vasilikos Terminal Services) and modelling the barriers women in science face (non-profit AIPFE Cyprus-Women of Europe). ESGIs started at Oxford University in 1968 and are now established in 25 countries. Participants from the School included numerous ECRs from the Applied and Computational Mathematics Group and the OR Group. We hosted an Innovate UK Artificial Intelligence in Health and Care Study Group with Industry three-day workshop (May 2019) with 35 researchers from the UK, strengthening academic awareness for potential research impact.

4.3 Collaboration and society

The societal outreach from Cardiff's mathematical research is extensive, yielding healthcare, educational and environmental benefits, together with societal advancement through public engagement. Formal activities include impact workshops with end users, development of educational resources and delivery of school outreach activities, teacher-training and public engagement events. In this short section we demonstrate outreach in (i) education and popularisation; (ii) local and global improvements to health and quality of life.

4.3.1 Education and popularization

Since REF2014 the School provided outreach events to over 5,000 secondary school pupils and developed a range of multimedia materials. For example, Woolley co-presents, edits and produces a biweekly podcast discussing mathematics and its representation in popular films. This is accompanied by supporting online blog posts with further links and puzzles. The website has had approximately 18,000 hits, with the podcasts consistently receiving 150 monthly listeners, including USA, Israel and China.

Kaouri was an invited judge at the science communication competition Famelab (2019 Cheltenham Science Festival). The contestants are science communicators from 27 countries who have won their national Famelab competitions. In July 2019 Kaouri was also a guest on a TV show in Cyprus where she discussed diverse applications of Mathematics in our lives. The School was involved in the organisation of the 'co-creation event with society' Breaking Barriers for Women in Science where 38 stakeholders from academia, business and policy-making in Cyprus came together and identified key issues hindering women's progression in STEM-related academia and business (October 2018).

Our researchers contributed to the OR Society's Learn About OR and were active in public engagement events including Soapbox Science, the cross-University STEM live event. Knight and his students are very active in PyCon UK's educational activities for children and teachers. Lewis spoke at the 2017 Hay Literary Festival on `The Mathematics of Networks and Maps'. Woolley was a Fellow of Modern Mathematics at the London Science Museum, where he helped redesign the mathematics gallery; he has also worked on `Dara O'Briain's School of Hard Sums'. Others contributed to live national broadcasts, most prominently number theorist Lettington (BBC1 'The One Show', Radio 4 'Today' Programme).

4.3.2 Health and quality of life

Two of our impact case studies are based on collaborations with the NHS. We also, however, conduct blue-skies research likely to yield significant future quality of life benefits.

One example is Leonenko's work on stochastic models for electroencephalogram imaging data, for children in a coma due to cerebral malaria. The development and calibration of these models has driven new theoretical results on fractional calculus, anomalous diffusion, Ornstein-Uhlenbeck-type and ambit processes, described in two very recent articles. The model parameters have been related to subsequent neurodevelopmental outcomes to predict which children will suffer the worst impairment. These predictions allow the prioritisation of cognitive rehabilitation for those in most need. This is joint work with neurologist Michael J. Boivin (Oregon), a former Fulbright scholar to DR Congo, who for two decades led major projects to address cognitive impairment in children due to diseases including HIV and forms the basis of a new collaboration (led by Leonenko) with statisticians at Imperial College London, supported by the DR Congo Ministry of Health.

A second important example is the new Cardiff-led H2020 DOWN2EARTH project on food and water insecurity in the Horn of Africa Drylands. Jones will apply his expertise in rainfall modelling to construct a flexible simulation model capable of delivering spatial-temporal fields of rainfall at high resolution (relevant to drylands); this will drive a hydrological model incorporating runoff, recharge of groundwater, soil moisture, and evaporation. The models will be sufficiently parsimonious to be fitted using reliably available data and will be implemented efficiently so that simulations can be used to account for climatic stochasticity. Model outputs will allow predictions of soil moisture and groundwater, yielding concise information that can be used by farmers and pastoralists, NGOs and governments to mitigate the impacts of climate change on rural livelihoods.

4.4 Global communication and dissemination

Cardiff mathematicians are enthusiastic communicators and regularly feature as guest speakers in the world's top institutes and conferences (Table 6). Several colleagues also hold editorial board positions (Table 7).

Beyond a substantial programme of events in Cardiff itself, of which the largest was the **2014 British Applied Mathematics Colloquium**, the global reach of our collaborations is particularly demonstrated by the extent to which we lead workshops throughout the world. Section 4.3 above mentions some events notable for their impact and the high level of research-user participation. Here, we highlight meetings we organised in some of the world's most important research venues, including the Isaac Newton Institute, Mathematisches Forschungsinstitut Oberwolfach and Institut Mittag-Lefller, together with events recognised as world-leading in their area:

i. The 2014 Mittag-Leffler Institute workshop *Modern aspects of the Titchmarsh-Weyl m-function and its multidimensional analogues* and the 2015 Oberwolfach workshop *Spectral Theory and Weyl Functions,* chaired by Brown.

ii. The 2016 Isaac Newton Institute six-month Operator Algebras and Subfactors (OAS) Programme, led by D.E. Evans.

iii. The 2016 Oberwolfach workshop on Quantum Field Theory, run by Lechner with Abdelmalek Abdesselam (Charlottesville), Stefan Hollands (Leipzig) and Christoph Kopper (École Polyechnique).

iv. The 2016 LMS-Durham symposium on Mathematical and Computational Aspects of Maxwell's Equations, organised by Brown, Yaroslav Kurylev (UCL), Marletta, Michael Plum (Karlsruhe) and Ian Wood (Kent).

v. The 2017 LMS-Clay Math Institute meeting on Microlocal Analysis and Applications, directed by Eswarathasan, Colin Guillarmou (Paris Sud) and Roman Schubert (Bristol)

vi. The 2018 RIMS-Glasshuku-style Seminar run in Kyoto by Woods with Tomoyuki Arakawa, Hiroki Shimakura and Hiroshi Yamauchi.

vii. The 2019 Isaac Newton Institute workshop *Estimation of entropies and other functionals: Statistics meets information theory*, run by Leonenko with Tom Berrett (Cambridge), Richard Lockart (Simon Fraser), Richard Samworth (Cambridge) and Yihong Wu (Yale).

viii. The 146th European Study Group with Industry in 2019, led by Kaouri.

ix. The 2019 Simons Center workshop on Operator Algebras and Quantum Physics, run by Lechner, with Stefan Hollands (Leipzig), Vaughan Jones (Vanderbilt) and Roberto Longo (Rome).

x. The 2020 ICMS workshop *Mathematical Sciences for Global Challenges Research Fund initiatives*, organised by a committee of 13 mathematicians including Harper.

Researcher	**Major research presentations**
Aliev	Uniform Distribution Theory, CIRM, Marseille, October 2018
Behrend	Galileo Galilei Institute for Theoretical Physics, Florence, 2015
	Oberwolfach, 2016
Ben-Artzi	Kinetic and Related Equations, BIRS-Oaxaca, 2015
Dirr	Durham Symposium on Homogenisation in Disordered Media, 2018
	Oberwolfach, 2018
Evans	BIRS-Banff, 2014
	Boole Centenary Meeting, Cork, 2015
	Oberwolfach, 2015
	60th Birthday Meeting of Fields Medallist Vaughan Jones, 2014
Kaouri	OCIAM30 (30th birthday meeting of OCIAM, Oxford)
Kadri	Ice-Fluid Interaction, Isaac Newton Institute, 2017
	Chinese Academy of Sciences, Beijing, 2018
Lechner	Oberwolfach, 2014, 2015, 2017
	Isaac Newton Institute, 2017
Leonenko	Fractality and Fractionality, Lorentz Centre, 2016
Marletta	Institut Mittag-Leffler, 2014
	Oberwolfach, 2015
	BIRS-Oaxaca, 2016
	OTAMP, Euler Institute, 2016
Mihai	70th Birthday Meeting for John Ball FRS, Oxford
Pennig	Fields Institute, Toronto, 2014
	Oberwolfach, 2015
	Institut Mittag-Leffler, 2016
	Isaac Newton Institute, 2017
Schmidt	Oberwolfach, 2015
Strokorb	Fields Institute, Toronto, 2016
	BIRS-Oaxaca, 2018

Wood	Vertex Algebras and Quantum Groups, BIRS-Banff, 2016
	OAS Programme, Isaac Newton Institute, 2017
	Vertex Operator Algebras and Symmetries, RIMS, Japan, 2018
	CFT and Related Topics, Chern Institute, China, 2019

Table 6: Examples of keynote talks delivered by staff

Researcher	Editorial Board
Aliev	Moscow Journal of Combinatorics and Number Theory
Harper	Founding editor-in-chief, Health Systems
Lewis	International Journal of Metaheuristics
Marletta	London Mathematical Society Editorial Board Mathematisches Nachrichten
Mihai	IMA Journal of Applied Mathematics; Journal of Elasticity; International Journal of Nonlinear Mechanics; Mathematics and Mechanics of Solids; Transactions of Mathematics and its Applications
Phillips	Journal of Non-Newtonian Fluid Mechanics Theoretical and Computational Fluid Mechanics
Strokorb	Journal of Extremes; Stochastic Models

Table 7: Editorial positions

Concluding remarks

The School expanded by more than 50% since REF2014, growing new international connections, expanding its portfolio of funders and stakeholders, and graduating PhDs into careers which reinforce its academic and industrial links. Our close collaborations with the School of Computer Science and Informatics establish a triangulation of expertise between OR and statistics, computer science, and pure mathematics, which will be rapidly enhanced when we move into our new shared building. Combined with our new directions in mathematical biology and strong links to engineering, this will leave us strategically placed to address the priorities set out in Section 1.9. These all focus on progressing topics of interest to the world scientific community. Attracting the world's best, as visitors and colleagues, will continue to be the most important mechanism by which we ensure success in fundamental enquiry, benefit to users, and globally connected research training for the next generation.