Institution: City, University of London

Unit of Assessment: 10 Mathematical Sciences

Overview

The Unit of Assessment (UoA) consists of the Department of Mathematics at City, University of London, as part of the School of Mathematics, Computer Science & Engineering. The UoA has strategically focused on three Research Groups undertaking fundamental research in pure and applied mathematics: in representation theory, mathematical physics and applied mathematics. These three areas of research are supported by our existing infrastructure, they are distinctive among our peers in the London area, nationally and internationally and their impact strategy has come to fruition.

The Department underwent substantial change before REF2014. While that period was characterised by growth and expansion as part of City's academic recruitment exercise, since then it has maintained its size, consolidated its position in areas of international importance and improved on its overall quality in all aspects. Staff members have enhanced their national and international esteem, the UoA's research output in the form of publications has further improved in quality and volume, the number of PhD students and postdoctoral fellows has increased by a factor of 2.5 and the UoA's research income in the form of research grants has more than doubled.

The UoA has maintained its size with current numbers of staff included in the submission at 15.6 full-time equivalents (FTEs) compared with 16.05 FTEs for REF2014.

1. Unit context and structure, research and impact strategy

The research strategy of the School is to develop internationally leading and sustainable research in selected distinctive areas of strength. In Mathematics the focus is on both pure and applied fundamental research in areas of current relevance.

A systematic staff recruitment process has attracted excellent staff, creating three internationally competitive Research Groups of global distinction with critical mass. Following REF2014, the Applied Mathematics Group has been created from the Mathematical Biology Group, two members of the Mathematical Physics Group and a new strategic replacement appointment. The purpose of this restructuring has been to assemble expertise in an area that leads to the generation of social economic impact. The Department has considerably expanded its doctoral and postdoctoral programme.

Further growth is planned, reaching 20–25 academic staff FTEs by 2025 within the existing infrastructure. The growth in academic staff numbers will be supported by a restructuring of the Mathematics undergraduate and graduate subjects taught currently in Engineering and Computer Science within the school. The strategic plan is that the relevant courses will be taken over by new and existing Mathematics staff, hence extending improvement in visibility for the Department and, in addition, increasing opportunities for collaboration and cross-fertilisation with the remaining part of the School. The expansion will strengthen the currently existing groups, further develop impact strategic areas and possibly lead to the formation of a new Research Group.

The membership, expertise and strategic aims of the Research Groups are as follows:

Representation Theory

Since REF2014, this group has become a major international centre for representation theory.

Membership: **Professor Joe Chuang** (Professor), **Dr Anton Cox** (Reader), **Dr Maud De Visscher** (Reader), **Professor Radha Kessar** (Professor), **Professor Markus Linckelmann** (Professor), one postdoctoral (Dr Xueqin Hu). In addition, in the current assessment period, the group had five Engineering and Physical Sciences Research Council (EPSRC) postdoctoral fellows (Dr Chris Braun, Dr John Enyang, Dr Chris Bowman, Dr Jorge Vittoria, Dr Florian Eisele), one Heilbronn fellow (Dr Neil Saunders), four completed and four active PhD students.

Expertise: The group has a broad range of expertise in mainstream modern representation theory. The research focus is on gaining deep conceptual understanding of algebraic, combinatorial, geometric and topological structure. The main areas of expertise are: finite dimensional algebras (all), symmetric groups and Hecke algebras (all members), representations of finite (Chuang, De Visscher, Kessar) and algebraic (Cox, De Visscher) groups, Brauer and other diagram algebras (Cox, De Visscher, Linckelmann), triangulated categories and dg categories (Cox, Kessar, Linckelmann), fusion systems (Kessar, Linckelmann), operads and homotopy algebras (Chuang).

Visions, objectives, strategy: The group plans to grow its role as a global leader in this research community through its high-quality research in a number of closely related, internationally active fields and through its organisational, educational, advisory and editorial duties. The group aims to continue to attract postdoctoral researchers through EPSRC grants (five separate three-year grants held in the current assessment period).

In joint work in 2013 Kessar affirmed one direction of Brauer's 55-year-old Height Zero Conjecture. Her work set the stage for a possible tackle on the other counting conjectures, e.g., Alperin's famous weight conjecture. The next five to ten years are expected to lead to significant successes in the area and the group at City expects to play a key role in this development and, more importantly, in setting the agenda for the subsequent five years, with a focus on homological methods to gain genuine understanding of the counting conjectures. This was the main motivation of the EPSRC grant “Integrable derivations and Hochschild cohomology of block algebras of finite groups” (ML, 2015–2018), relating the Morita equivalence classes of blocks and their fusion systems.

De Visscher and collaborators have developed a completely new approach to the famous Kronecker problem via one kind of diagram algebra, the partition algebra. Little progress had been made on the problem for a hundred years despite the attentions of many experts. The EPSRC-funded project “The partition algebra: a new approach to the symmetric group and applications to P vs NP” (2014–2017) and the Royal Commission of the Exhibition of 1851-funded project “A new approach to the Kronecker problem” (2013–2016) explored further the connections between classical representation theory and diagram algebras arising from Physics. This resulted in the combinatorial description of the largest-known family of Kronecker coefficients. The group hosted a major international conference, “Kronecker Coefficients Conference 2016” at City, accelerating the interactions between the Representation Theory and Computer Science communities and creating a platform for future development.

Cox and collaborators have developed a new approach to the graded representation theory of Hecke algebras using diagrammatic Cherednik algebras, which has given the strongest-known results in positive characteristic and suggests a new generalisation of the Lusztig conjecture to higher levels.

The group plans to continue its involvement in fields related to representation theory. The EPSRC grants “Homological algebra of Feynman graphs” (2012–2015) and “Derived localisation in algebra and homotopy theory” (2016–2019) explored connections between representation theory and homotopical algebra. For example, certain constructions in the modular representation theory of finite groups were reinterpreted as homotopy invariant noncommutative localisations. The goal is to gain further insight into the counting conjectures mentioned above from the perspective of homotopical algebra.

Mathematical Physics

With one new permanent appointment (Forini) and one departure (Caudrelier), this group has maintained its size and has consolidated its central role in specialised areas of quantum mechanics, quantum field theory and string theory. Two members with a research focus on fluid mechanics were reassigned to Applied Mathematics. This group has improved in all aspects, including quantity and quality of research publications and research grant income, and especially in the quality and overall volume of supervised PhD students, as well as in numbers of postdoctoral fellows.

Membership: **Dr Olalla Castro-Alvaredo** (Reader), **Dr Alessandro De Martino** (Senior Lecturer), **Dr Valentina Forini** (Lecturer, 01/2018), **Professor Andreas Fring**, **Professor**

Yang-Hui He, **Dr Bogdan Stefanski** (Reader), Dr Vincent Caudrelier (Lecturer 2007–2016), five postdoctorals and 21 PhD students. Specifically, four long-term externally funded postdoctoral researchers and visitors (Dr Wenhua Wang, Professor M H Y Moussa, Dr F Correa, Dr A Arda), one EPSRC-funded postdoctoral (Dr I M Szécsényi) and ten completed and eleven active PhD students.

Expertise: The group's research activities are concentrated on topics in quantum field theory, quantum mechanics and string theory. Extensive commonly shared expertise in various techniques and methods, developed originally in the context of classical and quantum integrable systems, creates a unique cohesive and vigorous environment. Castro-Alvaredo, Fring, De Martino and Stefanski have a strong track record in the form factor programme using exact scattering matrices where the ultimate aim is to compute physical quantities in a non-perturbative manner. While there are still some fundamental issues being considered, the main goal is now the application of these techniques. Castro-Alvaredo's focus in past years has been the application to the study of measures of entanglement and of the out-of-equilibrium dynamics of integrable models, whereas Stefanski applied these methods to modern aspects of string theory and Fring to new types of quantum systems. The research of Stefanski, He and recently appointed Forini is at the interface between mathematical and high-energy physics, with extensive expertise on gauge theories, string theory and stringy phenomenology. Stefanski is a leading expert on string theory, holography and its applications to low-supersymmetry settings. His expertise includes low-supersymmetry gauge theories, the relation between stringy phenomenology and algebraic geometry and pioneering the use of machine learning in the string landscape. The focus of Forini's research is on developing general methods for the quantitative study of the quantum behaviour of string and gauge theory models relevant in holography. Fring applied the techniques of integrable systems to the study of non-Hermitian systems with antilinear (Parity-Time reversal)-symmetry in classical systems, quantum mechanical systems and, more recently, in scalar field theories with non-Abelian gauge symmetries.

The group's theoretical expertise is complemented by work on applications in condensed matter physics. De Martino has extensively worked on the theoretical properties of graphene nanostructures. His seminal work establishing the possibility of confining electronic states in a monolayer graphene sheet by inhomogeneous magnetic fields is built on in a variety of scenarios. De Martino's recent work on spin-orbit interactions shows that graphene p-n junctions could be used as a building block in a spin-field-effect transistor and that the conduction properties at the sample's edges are severely affected when those interactions are not uniform.

Visions, objectives, strategy: Stefanski has pioneered the use of exact integrable methods for generic unprotected quantities in holography with low-supersymmetry. He will build on these findings to determine their implications for non-perturbative low-supersymmetry gauge theories. Forini is leading international efforts in using lattice field theory methods to analyse 2d string worldsheet models; she will significantly expand these tools and use them to extract exact information about large classes of strongly coupled gauge theories. Y-H He is a leading international expert on supersymmetric gauge theories, the gauge/string correspondence and algebraic geometry, and will investigate the new links between scattering amplitudes and geometry. A key component of future work will be the exploitation of recent results in work in settings with less supersymmetry. He is part of international collaborations that use state-of-the-art algorithms in computational algebraic geometry to sift through string vacua in search of the Standard Model. He has been pioneering Machine Learning tools in the study of the String Landscape. In collaboration with his Northeastern and Cornell colleagues, he will apply the latest AI techniques to data-mine the Kreuzer-Skarke dataset of some 500 million Calabi-Yau three-manifolds in order to precision chart the Stringy Landscape. Stefanski, Forini and He are ideally placed to produce significant new results in unravelling how the gauge/string correspondence works. They are at the forefront of some of the major recent developments and have the right combination of complementary expertise in string and gauge theories, as well as the necessary mathematical skills, to produce significant new advances.

Castro-Alvaredo will build on her earlier work that predicted universal features of entropy within the interpolation regime between the ultraviolet and infrared that were previously not accessible and that proposed a new method to access such features in a variety of theories and setups.

Since that work was disseminated this understanding has been extended to integrable discrete models (spin chains), critical but not conformally invariant theories, integrable theories with a boundary, non-unitary models and, most recently, to a wide class of excited states of integrable quantum field theories. In addition, Castro-Alvaredo has made important contributions in the development of a new technique to study the time-evolution of correlators in integrable models driven out of equilibrium through what is known as an inhomogeneous quench. This application combines hydrodynamics concepts with a quasi-particle description derived from the Bethe ansatz. This recent publication from 2016 has already attracted more than 332 Google Scholar citations and is considered foundational in the field. These works are particularly timely as integrable models at and out of equilibrium are now routinely realised in cold atom experiments. For the first time many theoretical quantities can be precisely measured.

Fring will build further on recent results obtained from applications of techniques developed in the context of integrable systems to non-Hermitian systems. New types of models with novel properties were found, some of which are elaborated on in a recent coauthored book *PT Symmetry in Quantum and Classical Physics* (World Scientific, 2019). Different types of results include a series of works on the extension to time-dependent quantum systems that, for the first time, have provided a meaningful physical interpretation of the spontaneously broken phase, predicting new unusual physical behaviour. For instance, for the entropy it was found that, unlike in any other regime, it becomes asymptotically stable, hence providing a regime of coherence in which quantum computers may be operated. Many more new effects are to be expected in this regime. Further existing results include work on non-Hermitian quantum field theories. Fring will continue to make contributions to this field, building on, among other approaches, his recent extension of Goldstone's theorem and the Higgs mechanism to non-Hermitian quantum field theories, with Abelian and non-Abelian gauge symmetry groups, well-defined BPS limits and field theories that possess Skyrmion solutions.

De Martino is continuing his line of research on fundamental aspects of the physics of graphene and other two-dimensional materials, the focus of which is on the properties of ideal mesoscopic devices hosting phenomena relevant for future electronic and spintronics applications. These now include twisted bilayer graphene nanostructures, a new form of graphene first discovered experimentally in 2018 and currently attracting a huge interest in the field. Moreover, De Martino is developing a new line of research on the fundamental mechanisms of electrical transport in two newly related classes of materials discovered recently, the topological insulators and the Weyl semimetals, which promise to be relevant for the future development of quantum computation. In this research deep mathematical ideas from the field of topology can be used to understand the origin of fascinating new physical phenomena observable in these materials.

Applied Mathematics
This group is newly formed by two members of the former Mathematical Biology Group (Baronchelli and Broom), new addition Noble and two former members of the Mathematical Physics Group (Kerr and Silvers) working in fluid mechanics.

Membership: **Dr Andrea Baronchelli** (Reader), **Professor Mark Broom** (Professor), **Dr Oliver Kerr** (Reader), **Dr Robert Noble** (Lecturer, 07/2020), **Dr Lara Silvers** (Senior Lecturer), Dr Anne Kandler (Lecturer, 2012–2016), one postdoctoral, nine completed and five active PhD students.

Expertise: The group applies mathematical methods to various fields in biology, the social world and fluid dynamics.

A significant area of research is the application of game theory to the mathematical modelling of biology (Broom, Kandler, Noble). Broom uses the concept of the Evolutionarily Stable Strategy (ESS), involving the development of fundamental theoretical work including multiplayer games and the evolution of specific animal behaviour such as dominance hierarchies (with Kandler), kleptoparasitism and biological signalling. A major recent theme of his work (including with Silvers) is the stochastic modelling of evolution on structured populations, including networks, involving multiplayer games. Kandler works on the modelling of processes of cultural evolution, in particular the processes of language dynamics and the dynamics of social norms. Noble uses mathematical and computational models to investigate the evolution and ecology of cancer, in

collaboration with experimental biologists and clinicians, in particular investigating how aspects of ecology shape evolutionary dynamics.

A second main area of research, with connections to the evolutionary models above, focuses on human behaviour, collective dynamics and network science (Baronchelli). Baronchelli's main interest is to quantify and model the dynamics of complex socio-technical systems. His research relies on mathematical and multi-agent modelling, data science and experiments with human subjects. Two major recent themes of his work include the emergence and dynamics of social norms and pioneering research on the cryptocurrency ecosystem (on the four axes of coding, usage, market and governance of cryptocurrencies).

The third main research area focuses on fluid dynamics (Kerr, Silvers). Kerr has developed a new approach for investigating the optimum linear growth of instabilities in evolving systems, including re-evaluating which are the most appropriate measures of the growth of such disturbances in evolving systems. These have been applied to longstanding problems such as heating fluid from isolated boundaries and the heating of salinity gradients from a vertical wall. Silvers' primary research area focuses on deepening our understanding of the solar interior. Recent work has centred on understanding instabilities that occur inside the Sun and also the interactions between buoyant magnetic structures and penetrative convection. Her research combines analytic and computational techniques to understand both the stability and evolution of systems

Visions, objectives, strategy: There is a strong focus on interdisciplinarity within the group with collaborations with fields such as artificial intelligence, biology, computer science and social sciences. The vision is to keep publishing world-leading research in the area of mathematics applied to biology, social systems and fluid dynamics and to further strengthen the leadership of the group in its areas of research in the UK by further developing activity in PhD student recruitment, grant capture and collaborations with top universities and industries worldwide. In particular, Broom's focus is continuing the development of realistic structured population models centred upon the new ITN grant (2021–2025). Baronchelli's focus is expanding the work on cryptocurrencies and helping to build a thriving community of researchers around this theme. By uniting mathematical, theoretical and computational approaches, Noble seeks to establish general principles of cancer evolution with applications to improving clinical forecasting and treatment. Kerr will apply his newly developed methodology to investigate the stability of convection problems whose properties have previously precluded rigorous use of traditional stability analysis. Silvers is focused on developing a rich understanding of the impact of temperature-dependent transport coefficients on dynamics in the solar interior. The group will maintain and further develop its strong interdisciplinary links across the university (Aeronautical Engineering Group, Artificial Intelligence Research Group, Centre for Mathematical Neuroscience and Psychology, Data Science Institute).

Communalities between groups: The Department has considerable breadth of expertise, but the three groups also share much common ground. Representation theory is of importance in many aspects of the research carried out by the Mathematical Physics Group. Y-H He from the latter group and Baronchelli from the Applied Mathematics Group have collaborated on a project investigating the Birch-Swinnerton-Dyer conjecture that describes the rational points on an elliptic curve, bringing expertise from number theory and Data Science together.

Open access: The Department is committed to supporting open access to its research and is fully aligned with the University's policy as outlined in REF5a. With 321 uploads since January 2014, the Department has one of the highest numbers of deposited articles in the University, with 141,384 associated full text downloads in the period from 30/06/2011 to 31/01/2021. In addition to making their publications available on the university-wide open access system, almost all staff members make all of their publications available on the arXiv repository of electronic preprints prior to publication, as is common in this field of research.

Impact strategy: Almost all research carried out by members of the Department is fundamental in nature, so that social economic impact is not generated as a natural by-product of the main lines of research as outlined above. However, in the process of the formation of the Centre of Mathematical Science and subsequent Department of Mathematics, some trained

mathematicians with more applied lines of research stayed in the Business School. For REF2014 the Department has relied on the expertise of these colleagues to produce high-quality social economic impact.

Since then, we have consequently pursued an impact strategy that leads to impact cases emerging naturally from the research carried out within the current setting of the Department. We have systematically strengthened and supported the members of the Department who carry out applied research. For this purpose we have also merged the former Mathematical Biology Group with two former members of the Mathematical Physics Group into the new Applied Mathematics Group. Staff in this area who left the Department have been replaced by researchers working in a similar applied field. Kandler has been replaced by Noble whose research is very likely to produce social economic impact as he uses mathematical and computational models to investigate the evolution and ecology of cancer in close collaboration with experimental biologists and clinicians.

Our strategy has come to fruition as the research of Baronchelli has led to various impact cases, of which one, entitled “Combating cryptocurrency illicit activities via market analysis”, is submitted to REF2021. The case grew naturally out of two general lines of research pursued by Baronchelli: his decade-long research in Network Science on modelling of temporally evolving networks and a more recent activity focused on the analysis of the cryptocurrency ecosystem. Analysing transactions in the Dark market, the study revealed that a broader, systemic, focus is crucial to combat illegal activities funded by cryptocurrencies and the importance of considering longer chains of bitcoin transactions. Chainalysis, a software company that supplies the world’s leading banks, businesses and governments, has taken the message of this research on board by embedding both practices in its products. Thus, the impact relates to the understanding of the financial markets affecting the analysis and behaviour of industry players.

Staff engagement in impact activities is recognised in our departmental workload scheme and in the promotion process. At the School level, impact is supported by running an annual Higher Education Innovation Fund (HEIF) competition open to all staff. In total, £75,000 is available annually across the School, with staff being able to bid for up to £10,000. The fund is intended to help City academics demonstrate the wider impact of their research beyond academia by facilitating relevant academic enterprise activities. Administrative support for impact is provided by two specialist impact officers who work across five schools.

The research activities in the Applied Mathematics Group also led naturally to knowledge transfer activities. Baronchelli is engaged in a Knowledge Transfer (KT) project entitled “Beryl – data science for smart bike sharing” supported by a £278,074 grant. This project fits well into his line of research activities on human mobility and the modelling of human behaviour. It is jointly carried out with Beryl, a leader in cycle safety equipment that has recently launched its own bike-sharing schemes in a number of cities in the UK. The CITY-Beryl Knowledge Transfer Partnership (KTP) aims to develop and license a unique AI-driven data analysis, modelling products to reduce the core operating costs of bike-share operators: principally bike redistribution and bike maintenance.

In response to COVID-19 members of staff have also partially redirected some of their research activities to contribute to the tackling of the pandemic. Baronchelli has started a new project that monitors Dark Web trading in COVID-19-related goods and services entitled “COVID-19: monitoring the effects of the pandemic on illicit online trade” supported by a £272,813 research grant. The first results of this research were published in August 2020 on the open-access pre-print repository arXiv. Y-H He is using his expertise in applications of the Susceptible-Exposed-Infected-Recovered-Dead (SEIRD) differential model that serves for the analysis and forecast of the COVID-19 spread. The research is ongoing with results due to be published early in 2021.

Given the nature of research carried out in the Applied Mathematics Group it is likely that similar impact success stories will be developed by other members.

2. People, including:

a) Staffing strategy and staff development

Within the context of the University's staffing policy as outlined in section 3 of the REF5a document, the Department seeks to create an environment that encourages all members of staff to conduct research at the highest level, through publishing in leading international journals, applying for and gaining research grants, collaborating with leading international researchers within their fields, receiving research invitations to other institutions and receiving invitations for conference presentations and keynote addresses. The following mechanisms are in place to enable and support staff to achieve these expectations.

Changes: Since REF2014, two new permanent FTEs have been made – Forini and Noble – replacing the two permanent FTEs Caudrelier and Kandler in the same area of research. Three non-permanent full-time researchers, Dr Benjamin Favier (1.0 FTE), Professor Jiri Mathon (0.2 FTE) and Professor Carl Bender (0.25 FTE) have left. The number of PhD students and postdoctoral fellows has risen from 10 to 40.2 and 7 to 13, respectively.

With these changes two of the Research Groups have kept their necessary critical mass and structure constant to carry out internationally leading research on a sustainable long-term basis. The Applied Mathematics Group has been newly created out of the remaining members in the former Mathematical Biology Group (Broom, Baronchelli), with one replacement (Kandler → Noble) and two members previously assigned to the Mathematical Physics Group (Kerr, Silvers) who both focus on applications of fluid mechanics.

Support: The Department supports applications for fellowships that sustain teaching replacement. Over the last few years, the structure and teaching of degree programmes has been optimised, offering staff more time for research. This process has led to a reduction in the teaching load from typically three modules per year to two modules per year. (A 15-credit module typically consists of 30–40 lecture hours.) This streamlined teaching load allows researchers to arrange their entire teaching in one term only so that the remaining term may be used as a short sabbatical. During term-time colleagues cover teaching for each other to allow for absences to enable focus on research at key stages. Short absences of one week are typical, but occasionally arrangements for longer periods of absence are made. For instance, He and Stefanski have used this covering scheme for a period of a half-term of absence each to enable an extended visit to the internationally renowned Perimeter Institute for Theoretical Physics in Waterloo, Canada, and other institutions.

Staff may apply for sabbatical leave for a period of time up to a maximum of one-seventh of service at the University to support more sustained activity, with applications being approved by the School Board of Studies and Senate. Successful applications falling into the period of assessment were made by Broom, Castro-Alvaredo, Chuang, He, Kessar and Linckelmann, resulting in establishing further links to international institutions especially in Europe (Bologna University, Italy) and the USA (University of California, Berkeley; University of North Carolina).

Newly appointed members are given reduced teaching and administration loads, typically no more than one module in the first year for new lecturers. The Department has a contribution model that keeps track of staff teaching and administration loads to maintain an equitable distribution of duties.

The Department provides a research fund of £5,000 per year to support visitors for seminars and research collaborations. Part of the overheads from research grants, ~£40,000, has been used to support staff and PhD students for conference visits and to partially finance PhD studentships. Typically, each member of staff is given £1,000 per year for travel.

The Department creates a family-friendly atmosphere. All our committee meetings, seminars and other departmental events are scheduled during core hours. Flexible working hours are supported, with staff given the opportunity to indicate availability in the design of their timetables. City offers maternity leave entitlement for a full year in two formats, to enable staff members to choose an option to suit their individual finances. Silvers and Forini made use of this scheme during the assessment period.

Guidance: New staff are assigned a mentor who guides them through their probation period and beyond. Senior staff provide advice on all teaching and research matters, such as, for instance, informal feedback on grant applications or draft versions of publications. They are also equipped with new computers and are entitled to apply for financial support, typically £5,000, from a University Pump-Priming research fund, which is designed to assist early-stage projects with the potential to attract external funding. Recent successful examples include Baronchelli, who used it to establish a new research collaboration with Professor Centola, a leading Social Scientist in the Annenberg School at the University of Pennsylvania, currently ranked as the first School for Communication in the world, and Forini who is using the funding to employ a PhD student to assist her in developing codes for numerical computations needed for her current research project.

Promotions: To aid staff development and ensure a fair promotion procedure, all staff are appraised annually by a senior academic within their respective fields of research expertise. For this purpose a standard academic CV has been developed to enable staff to provide information about their academic achievements on a consistent basis across the University. During the assessment period DeVisscher and Baronchelli have been promoted to Senior Lecturer and subsequently to Reader in Mathematics, Castro-Alvaredo to Reader in Theoretical Physics and He to Professor in Mathematics.

b) Research students

Recruitment: The Department has recruited over 40 highly qualified PhD students from top institutions around the world, among them the Universities of Oxford and Cambridge; Imperial College London; King's College London; University College London; University of Bologna, Italy; Ludwig Maximilian University of Munich, Germany; Tata Institute of Fundamental Research Mumbai, India.

Studentships were widely advertised on websites and via networking with research colleagues. An efficient fast-tracking procedure has enabled the Department to attract some students ahead of other leading institutions. The number of applications has risen substantially each year so that the Department is in a position to select from the very best students with suitable backgrounds from around the world. During the period of assessment **22.5** doctorates have been awarded compared with 10 during the REF2014 period:

year	2013/14	2014/15	2015/16	2016/17	2017/18	2018/19	2019/20
PhD awards	4	3.5	2	5	3	1	4

The overall quality of the theses produced by this new cohort of students has improved when compared with previous years, as well as students' success rate in continuing in academic careers. Many of these students have moved on to embark on accomplished research careers in other institutions. For instance, Dr Sanjib Dey, who completed in 2014, has taken up postdoctoral positions at the University of Montreal, Canada, the prestigious Institute des Hautes Études Scientifiques (IHES) in France and is currently on a long-term contract at the INSPIRE Faculty at IISER Mohali, India. Dr Niamh Farrell completed her PhD in 2017 and has worked since then as a postdoctoral researcher at the Technische Universität Kaiserslautern in Germany. Dr Veronika Witzke concluded her PhD in 2017 and is now a researcher at the Max Planck Institute for Solar System Research in Germany. In 2020 Dr Julia Cen joined the prestigious Los Alamos National Laboratory in New Mexico, USA, for postdoctoral studies.

Currently there are **20** students in the Department actively carrying out research and studying for their PhDs.

University and School support: In addition to grant income, research in Mathematics has benefitted since REF2014 from substantial University investment resulting in 18.5 University or School-funded PhD Studentships worth £1,137,750, two Pump-Priming grants and several additional PhD fee waivers. The School has spent a total of £19,421,254 on infrastructure. Research is also promoted through various School and University-wide annual research competitions. To encourage potential social economic impact and KT activities, additional Studentships have been made available since 2019, for which the School matches half of the funding when the other half is provided by industry. Staff maintain personal accounts that are supported by an eight per cent contribution from grant income.

Research student training: Students are assigned to a lead and second supervisor within a Research Group. They are strongly encouraged to widen their horizons by attending our weekly departmental research seminars in-house and at institutions in London, such as the London Triangle Seminars and the London Integrability Journal Club, focusing mainly on aspects of string theory, and the London Algebra Colloquium. Specialised courses taught by mathematicians from universities across London are delivered by the London Taught Course Centre (LTCC). Each year the centre offers around 18 basic courses and 15 advanced courses, each with a typical length of 10 hours. Students are expected to attend 3–4 courses per year, ensuring a minimum of 100 hours' training during their studies, and are required to take an examination at the end of the year. City staff are regularly involved in the teaching of these courses with Kerr and Linckelmann being engaged during the assessment period. In-house, students may attend relevant lectures provided as part of the MMath programme and the MSc in Decision Sciences that run during the assessment period. Students participate in theme-oriented study groups and Journal Clubs, where, after the first year, they present their own work in a seminar to the Department. In addition, they run their own study group and PhD student colloquia that are financially supported from the Departmental research budget. Led by Stefanski, a new joint initiative with King's College London, The London Theory Institute, started in 2020, where staff from both institutions are teaching short specialised courses in Theoretical Physics with the aim to bridge the knowledge gap between the MSc and PhD level. All staff members from the Mathematical Physics Group are involved in delivery of these courses.

Students are also strongly encouraged to present their work at national and international schools, workshops and conferences. Support for this is provided by research grants and the Department's travel budget. After the first year the Department offers the opportunity for PhD students to gain teaching experience and improve their communication skills by tutoring undergraduate students. Assignments depend on progression and are coordinated by the Head of Department in liaison with supervisors.

Student progression is closely monitored by the School's senior tutor for research through annual progress reports and by means of a software system (the Research and Progress platform), whose use is mandatory for both supervisors and students. The system involves PhD students in the management of their own research projects with a flexible approach according to individual student needs. Key milestones are recorded, with required reporting in at least four meetings per term, an initial six-month report, an annual progress report, details of progression from MPhil to PhD status and intention to submit and transfer to writing-up status no later than year four. In addition, the University provides institutional support for PhD students as outlined in the REF5a document.

c) Equality and diversity

The Department is committed to the principle of equality in all processes as, for instance, reflected in the numbers of female and international staff, with 31 per cent and 69 per cent respectively. The proportion of female staff has slightly increased since REF2014.

City's strong commitment to Equality, Diversity and Inclusion (EDI) was recognised by a Bronze Award from Athena SWAN in November 2016, as outlined in the REF5a document. In October 2019, the School of Mathematics, Computer Science & Engineering received their own Athena SWAN Bronze Award. Two members of staff from the Department, Cox and Castro-Alvaredo, were actively involved in the preparation of the submission, which was characterised by the awarding panel as a "particularly strong submission", commending the School and the self-assessment team (SAT) for their "detailed, thoughtful and thorough work to address gender inequalities". The implementation and progress of the Athena SWAN action plan is overseen by our School's EDI committee, which is chaired by Silvers and has Castro-Alvaredo as a member.

Several current female members of staff take part in activities that promote women in science, in particular in mathematics and physics. As part of the annual global celebration of women in science and technology a regular annual event is hosted at City University on Ada Lovelace's birthday. Castro-Alvaredo and Forini were guest speakers in 2015 at "Ada Lovelace Day: Are Women in Britain Frightened of Maths?" and in 2018 at "Career Progression of Women in STEM: Challenges and Opportunities", respectively. Forini is a member of Gender in High

Energy Theory (GenHET), a permanent working group hosted by the CERN Theory Department, whose main objectives are to monitor the situation of women in High Energy Theoretical Physics, increase the awareness of gender issues in the field, improve the visibility and presence of women in decision-making roles and provide networking and mentoring support, particularly to early-career researchers. Forini appears in a podcast series "Ask Different" (episode 2) from the Einstein Foundation Berlin talking about her career path and the need for more female role models in top-notch science. Forini has also mentored female PhD students and postdoctoral scientists within the program "*Frauen in den Naturwissenschaften am Campus Adlershof (FINCA)*" [Women in Science in the Campus Adlershof] at Humboldt University Berlin and continues the activity of mentoring during the "Emmy Noether Workshops" held regularly at the Perimeter Institute for Theoretical Physics in Waterloo, Ontario, Canada.

3. Income, infrastructure and facilities

The Department is committed to expanding its current track record of successful applications for research, travel and conference grants, building on the experience of new staff and through collaborations and joint grant applications with colleagues from other Schools in applied research areas. For instance, links to the Department of Computer Science exist in the form of joint PhD supervisions by Broom and He. Broom has collaborated with Professor Collins of the City Law School on game-theoretical approaches to investment treaty arbitration.

The School and University Research Office and Enterprise services provide support for identifying and accessing relevant funding opportunities as well as help with the submission of high-quality proposals (including costing) and subsequent assistance with the management of successful grant applications. The Department's total grant income during the assessment period has increased from £1,243,000 during the REF2014 period to **£1,923,714.80**.

The distribution by year and HESA funder category is summarised in the following table:

(in units of £1000)	2013/14	2014/15	2015/16	2016/17	2017/18	2018/19	2019/20	∑
EPSRC	143	146	354	276	329	134	87	**1470**
ESRC							15	**15**
STFC	13	-4	28	24	11	20	12	**103**
UK charities (open)			80	-60		-7	40	**53**
UK charities (other)	32	55						**88**
UK government				59	8	12		**79**
UK industry						9		**9**
EU government			11	12	24	44		**92**
Non EU (other)						11	4	**14**
∑	**189**	**197**	**473**	**312**	**372**	**223**	**158**	**1924**

In addition, the Department had consultancy income of £3,100 in 2017/18.

In general, desktop computers and software for research are financed by personal grants; the School IT committee provides funding for complete replacements of computers every five years.

Journal subscriptions, in the areas represented by the three Research Groups, and specialist books are financed through the School's library fund. In addition, the Department runs a voucher scheme for staff and PhD students to purchase journal articles through City's electronic inter-library loan system. The central library subscriptions to Scopus, the Web of Knowledge and MathScinet allow access to abstract and citation databases of peer-reviewed literature.

As exemplified by the investment in the promotion of staff, funding of PhD studentships, etc., the Department has also benefitted from strategic investments arising from both University and School

Strategic Plans, enabling further development in research. Excellent academic and research staff are the Department's key assets.

4. Collaboration and contribution to the discipline or research base

Guests, seminars, workshops and conferences

The Department runs a weekly mathematical research seminar series covering topics from its three areas of expertise. This serves as an effective mechanism for establishing interaction between the Research Groups. Since REF2014 the Department has hosted more than 150 speakers from the UK, Europe, the USA, China, India, Brazil and Canada. Seminar speakers have included eminent researchers, Fellows of the Royal Society and postdoctoral researchers who are recent entrants to their field.

The Department has hosted numerous workshops and conferences, strengthening collaborative research and contributing to internationalisation and dissemination of findings. During the assessment period these have included:

- 67th, 70th, 73rd, 75th, 76th and 80th Bristol Leicester Oxford Colloquium (2015–18) (~30 participants)
- The London Algebra Colloquium (2016, 2017, 2018) (~40 participants)
- Data Natives (2015, 2016, 2017, 2018, 2019) (~80–100 participants)
- Data Beers (2x2015, 2x2016, 2x2017, 2x2018) (~200 participants)
- Homological Methods in Algebra, Geometry and Physics (2014) (~40–50 participants)
- 3rd South East Mathematical Physics Seminar (2014) (~40 participants)
- Entanglement Entropy in Many-Body Quantum Systems (2014) (~52 participants)
- Mathematical and Physical Aspects of Biology (2015) (~35 participants)
- Mathematical Models in Ecology and Evolution (2017) (~230 participants)
- Conflict, Competition, Cooperation and Complexity workshop (2017) (~35 participants)
- 12th South East Mathematical Physics Seminar (2018) (~25 participants)
- The 6th ECR meeting of the South East Mathematical Physics Seminar (2019) (~30 participants)
- Applied Geometric Mechanics (2019) (~40 participants)
- Symmetries in Physics (2019) (~30 participants)
- Complexity and Token Economy (2020) (~150 participants)
- Cancer Adaptive Therapy Models workshop (online, 2020) (~130 participants)
- Seminar series on Pseudo-Hermitian Hamiltonians in Quantum Physics (online, 2020–) (~180 participants)

Members of the Department have served as organisers and members of Organising Committees of more than 70 international conferences and workshops.

While most planned workshops and conferences had to be postponed due to COVID-19, several members of staff took the initiative to engage in new activities. Fring launched a new weekly/biweekly Virtual Seminar Series on Pseudo-Hermitian Hamiltonians in Quantum Physics (vPHHQP) in April that has a full programme until the summer of 2021 and is planned to run for at least a further year. It has attracted an international audience of more than 180 registered participants with rising interest. Based on the success of the series Fring is coediting proceedings with articles based on the online talks to be published in the *Journal of Physics: Conference Series* (IoP Publishing). Stefanski is running the London Integrability Journal Club jointly with members from King's College, which went online at the end of March 2020 and has since attracted a global audience of around 100 participants. Noble was involved in organising an international online workshop in December 2020, Cancer Adaptive Therapy Models, which attracted 130 participants.

The Department has extensive international connections through long-term visitors, postdoctoral fellows and external PhD students, who both benefit from the Department's expertise and enhance its research environment. The Department hosted around 40–50 long-term visitors during the assessment period (staying between three months and one year) and maintains active links, in the form of coauthoring papers and seeking funded research grants, to more than 150 universities and research institutions worldwide.

Further contributions to the discipline – publications, keynote speeches, editorial work, prizes

All members of the Department act as regular reviewers for scientific journals, evaluators of research proposals, external examiners for research degrees and session chairs for international conferences. Further key contributions to the discipline in the assessment period include the following.

The Representation Theory Group contributed 62 publications to its field. Members gave 49 invited keynote speeches at international conferences and lecture series in ICRA (Syracuse NY), EPFL (Lausanne) and Cambridge. Editorial work in journals include Chuang: Editorial Advisor LMS Journals (2011–2016); Member of editorial board, *Journal of Algebra and Its Applications* (2016–); Kessar: Consulting Editor, *Proceedings Edinburgh Mathematical Society*; Joint Chief Editor of the *Proceedings of the LMS* (2016–2018), Editorial board memberships: *Journal of Group Theory*, *Algebras and Representation Theory*, *Transactions and Memoirs of the American Mathematical Society*; Linckelmann: Editorial board member *Journal of Algebra* (2017–). All members of the group are regularly invited as main speakers at major international events and as visiting academics at world-renowned institutions. For example, Kessar and Linckelmann were invited professors at the Mathematical Sciences Research Institute, Berkeley (January–May 2018, Kessar held the Simons professorship). Chuang, Kessar and Linckelmann were invited participants at the Isaac Newton Institute, Cambridge (January–June 2020, Kessar will held a Simons fellowship). Chuang, Kessar and Linckelmann are organisers of the long-running series of conferences on Representations of Finite Groups in the Mathematical Research Institute Oberwolfach. Kessar was an organiser of a research semester at the Ecole Polytechnique Federal Lausanne (June–December 2016). Kessar is a member of the programme committee of the Joint AMS-SMF-EMS International Meeting in 2021. Linckelmann is a member of the scientific steering committee of the British Mathematical Colloquium. Chuang is a member of the advisory committee that selects areas of priority for the additional funding £300m made available to Mathematical Sciences as announced on 27/01/2020.

The Mathematical Physics Group contributed 138 publications to its field with 1,399 Scopus citations. Editorial work in journals includes Castro-Alvaredo: Editorial board member, *Journal of Physics Communications*; Fring: Advisory board member, *Journal of Physics A*; Editorial board member *ISNR Mathematical Physics, MDPI Universe* (2020–); He: Editor in Chief, *Journal of Modern Physics*; Editorial board member, *Mathematics MDPI* (2014–), *Frontiers in Mathematical Physics* (2014–); Special Collection Editor, *Journal of Clifford Algebras & Applications* (2020–). Fring will host the major conference "Pseudo-Hermitian Hamiltonians in Quantum Physics" in 2022 at City. The significance of this research is reflected in the fact that *Nature Physics* selected the application of Parity-Time reversal-symmetry to optical systems as one of the ten most important discoveries in physics in the period from 2005 to 2015.

The Mathematical Biology Group contributed 107 publications to its field with 850 Scopus citations. Editorial work in journals includes Baronchelli: Associate Editor of *EPJ Data Science* (2018–), *PLOS One* (2018–), *Frontiers Blockchain* (2018–); Broom: Editorial board member of *Journal of Theoretical Biology*, *Dynamic Games and Applications*, *Journal of Dynamics and Games*. Baronchelli is a visiting researcher at the Alan Turing Institute, being their lead in the Economic Data Science theme. His publications include articles in top interdisciplinary journals such as *Science* and *PNAS (2)*, with one paper submitted to the 41st Annual Meeting of the Cognitive Science Society (2019) being awarded a Computational Modeling Prize in Applied Cognition. Broom was the coordinator of an 11-university international consortium for the project "Conflict, Competition, Cooperation and Complexity: Using Evolutionary Game Theory to Model Realistic Populations" funded by the EU, which led to a 30-partner EU Innovative Training Network grant commencing in 2021. Baronchelli was the recipient of the 2019 "Young Scientist Award for Socio- and Econophysics" of the

German Physical Society (DPG), awarded for "outstanding original contributions that use physical methods to develop a better understanding of socio-economic problems". The group has won external grants from a variety of funders (Alan Turing Institute, ESRC, EU, Innovate UK, National Cancer Institute (US), STFC, US Army Lab) and provides consultancies to industry in the UK and abroad.

Outreach

Most members of the Department are involved in school visits and some participated in events where schools have visited City for a Mathematics taster day. Most of these activities are aimed at recruitment and are not particularly targeted at girls or any other demographic. However, in the case of schools that visit us, this is part of the "widening participation" scheme the University is involved with and so the schools often have students from disadvantaged backgrounds. It is worth noting that five female members of staff have been involved in these activities and can be seen as acting female role models for the girls that attend these events.

The activities of the Applied Mathematics group have been covered in the media including: *El País*, *Le Figaro*, *MIT Tech. Review*, *Scientific American*, *The Economist*, *The New York Times*, *The Washington Post* and at The World Economic Forum. Baronchelli wrote an invited Op-Ed for *Le Monde* (June 29, 2018) and his analysis on Twitter posts on the Brexit referendum was reported in Wallstreet-online.de. He was featured in *Science* for his research into the use of artificial intelligence for speeding up certain calculations used by physicists and mathematicians.

As a further regular outreach activity six members of staff, Baronchelli, Castro-Alvaredo, De Martino, Fring, He and Stefanski, take part each year in the "Pint of Science" initiative whereby Mathematics and Science talks are delivered at chosen pubs throughout London for the general public. Our PhD students have played a crucial role in the logistics and organisation of these events.