Institution: Keele University

Unit of Assessment: UoA10 Mathematical Sciences

1. Unit context and structure, research and impact strategy

Research Groups

The Mathematics Unit at Keele is renowned for its world-leading research in the general area of continuum mechanics and its interdisciplinary applications. It is also strengthened by its complementary expertise in algebra, logic and statistics. The continuum mechanics research is carried out by ***Solids*** and ***Fluids*** groups. The Solids group consists of six members: Prof C.J.Chapman (CJC), Prof Y.Fu (YF), Prof J.Kaplunov (JK), Dr M.J.Nieves (MJN), Dr D.Prikazchikov (DP), and Dr L.Prikazchikova (LP). The Fluids Group has five members: CJC, Prof J.J.Healey (JJH), Dr D. Lucas (DL), Dr S. Naire (SN), and Prof V. Shrira (VS).

The ***Solids group***, one of the largest in the UK, contributes to a broad range of challenging topics, including multiscale modelling of advanced materials and structures, large elastic deformations and structural acoustics. The group's focus is on basic research addressing modern industry's needs in developing novel nano-, meta- and soft materials, and layered and multicomponent structures. The ***Fluids group*** tackles a diverse range of fundamental problems of immense practical importance, including laminar-turbulent transition, flow with phase transitions, flow in porous media, geophysical flow, and aircraft and submarine noise. Areas of application of the group's research include most known fluid behaviour in nature and technology. The methodology underpinning and uniting research in the Fluids and Solids groups is mainly based on asymptotic approaches, and also involves numerical modelling and processing of experimental data. There is also an important synergy in the interests of both groups related to biomedical applications and wave propagation.

The ***Algebra & Logic*** and ***Statistics*** themes are represented by Dr P.Fletcher (PF), Dr P.Truman (PT) and Dr J.Cheng (JC), specialising in intuitionistic analysis, algebraic number theory, and financial statistics, respectively. The Unit is also involved in collaborative research on medical statistics (Dr J.Belcher, JB) and mathematical education (Dr D.Bedford, DB). Although JB and DB retired in 2019, they maintain strong links with the Unit, as does a renowned expert in Solid Mechanics, Prof G.A.Rogerson (GAR), who also retired in 2019. Prof M.Heckl (MH), specialising in structural and thermoacoustics, was affiliated with the Unit until 2016.

Review of 2014 plan

Since REF 2014, the unit has grown by 53% (from 8 to 12.25 FTE). The number of PhD students with passed vivas has more than doubled (from 9 to 20). The number of international visitors has risen dramatically, and now exceeds one hundred.

Developments in the key research areas prioritised in the REF2014 strategic plan resulted in significant progress of the *Solids group* in the modelling of advanced materials, including the derivation of a novel asymptotic formulation for flexural meta-surfaces (JK et al., Proc. Roy. Soc. A 2019), and new insights into localisation phenomena in soft materials (YF et al, J. Mech. Phys. Solids 2016, 2019, 2020). Other important achievements include development of a powerful methodology for incorporating gyro effects into analysis of lattice and flexural structures (MJN et al., Phil. Trans. Roy. Soc. A 2019), and crucial insight into the effect of boundary layers in non-local elasticity (JK et al., Proc. Roy. Soc. A 2016). The *Fluids group* advanced research in remote sensing of the ocean interior: a new class of sub-inertial internal waves extremely sensitive to the upper ocean stratification has been found and employed for remote probing of the upper ocean (VS with P.Forget, J. Phys. Ocean. 2015). Both groups produced important developments in various aspects of bio-medical modelling (e.g. SN et al., J. Tissue Eng. 2019).

In parallel, building upon Unit's historic strengths, ground-breaking work was continued in other areas. A novel conceptual framework for nonlinear waves trapped by oceanic jet currents, and, in particular, a new model of rogue waves, has been proposed (VS with A.Slunyaev, JFM 2014;

Phys. Rev. E 2014). A century-old stability problem for a stratified boundary layer has received a new and unexpected turn: it has been discovered that the neutral curves form a self-similar fractal structure, which has not been found previously in hydrodynamics (JJH, JFM 2019). There has been also a major advance in addressing the profound problem of how the instabilities of stratified shear flows evolve first into coherent structures and then into turbulence, and how re-laminarisation unfolds (DL et al., JFM 2017, 2019; DL with C.Caulfield, JFM 2017). Analysis of a new class of stability problems concerned with the dynamics of a surfactant-laden drop resulted in the discovery of an unexpected type of fingering instability at the drop's leading edge (SN with J.Goddard, JFM 2015).

A general multi-parametric approach for evaluating the lowest vibration modes typical of high-contrast multi-layered and multi-component structures has been established (JK, DP and LP, Int. J. Solids Struct. 2017; JK et al., J. Sound Vibr. 2019). Robust explicit formulations for surface, interfacial and edge waves have been further developed (JK and DP, Adv. Appl. Mech. 2017). A fresh analytic treatment of harmonic waves in curved elastic structures has been found, and new analytic results for elastic waves in a layer have been obtained using a novel representation (CJC with S.Sorokin, Proc. Roy. Soc. 2016, 2017). An analytical method for analysing period-doubling secondary bifurcations has been proposed (Fu with Z.Cai, SIAM J. Appl Maths 2015). Novel insight into dynamic failure phenomena in structured media has been put forward (e.g. MJN et al., Q. J. Mech. Appl. Math. 2019).

The achievements of the Unit are not confined to Mechanics. Within the *Statistics theme* a theoretical framework based on extreme value theory has been proposed for managing default exposure in futures markets (JC et al., Eur. J. Finance 2016). The *Algebra & Logic* theme produced important results on the Hopf-Galois module structure of algebraic integers in non-normal extensions of global fields (PT, J. Pure Appl. Algebra 2019), and Brouwer's weak counterexamples to classical theorems (PF, J. Phil. Logic 2020).

Interdisciplinary Research

From its inception Keele has pursued interdisciplinarity as one of its key guiding principles, with a current focus on *Sustainability and Health*. A strategic Faculty of Natural Sciences (FNS) initiative promoting materials aligns with the specialism of the *Solids group*. Most of the Unit's research maintains an explicit interdisciplinary spirit, built upon a solid mathematical core, with applications in engineering, geophysics and biomedicine, through joint efforts with academic and non-academic partners, in the UK and overseas. Examples include:

- Internal collaborations with the School of Geology, Geography and Environment (GGE) and Institute for Science & Technology in Medicine (ISTM) have a long and successful history of applying mathematical techniques to such diverse topics as tsunamis, lava flows, and bioreactor and aneurism modelling. Collaboration with *GGE* resulted in a breakthrough in developing a unique non-invasive technique of finding the run-up of palaeo-tsunamis, applicable to tsunami mitigation (VS et al., Pure Appl. Geophys. 2015).
- A substantial research programme by Keele mathematicians and psychologists exploring novel approaches of teaching STEM subjects has been carried out within the *Stoke-on-Trent Mathematics Excellence Partnership*.
- Industrial collaborations include theoretical underpinning of lightweight design for "green" cars (*TPV*) and modelling of dynamic forces in freight trains (*Amsted*).
- International collaborations with non-mathematical institutions involve academics from Canada, China, Denmark, France, Germany, India, Israel, Italy, Russia, Slovenia, Turkey, and US.

Research Strategy

Mathematics Research is embedded within the Faculty and contributes fully to the University's research strategy. The Unit's present and future strategy, ensuring its vitality and long-term prosperity, is built upon its outstanding expertise in addressing fundamental problems in continuum mechanics, grounded in real world challenges. The main objective is to produce world-leading research that addresses a diverse range of important issues and challenges in the natural sciences, engineering and medicine.

Aligned to FNS priorities, the *Solids group* will continue to focus on multiscale mathematical modelling of the static and dynamic behaviour of nano-, meta- and soft materials and structures. The high-tech industry of such materials is currently growing at breakneck speed, continuously posing challenging questions on their vibration and instability behaviour that can only be understood by advanced and novel mathematical analysis. The *Fluids group* will concentrate on flow instabilities and laminar-turbulent transition, where it has prepared the ground for major breakthroughs; and, in aeroacoustics, on the noise generated by contra-rotating aeroengine designs which is of great prospective importance because of their fuel efficiency.

In line with Keele's focus on *Health* and *Sustainability,* the joint efforts of both groups will also focus on biomedical modelling (soft biomaterials, bioreactors and blood vessels) and geophysical applications. At the same time, in addition to the areas of established record, opportunities in other fields will be pursued. Two recent key appointments (after the census date) in mathematical data science provide excellent prospects for integrating machine learning and mechanics in growing collaboration with computer scientists within the School.

The unit will continue to cultivate many local, national and international collaborative networks, thus enhancing greatly its research and PGR training capability. It will also ensure that the current healthy stream of UK and overseas PhD students is sustained. The proven flexible approach (UKRI, EU, industry, etc.) to securing external funding supporting strategic priorities will be retained. The unit will also remain proactive in organizing research events (e.g. *Euromech Colloquia*), making use of Keele's excellent infrastructure.

Facilitating Impact from Research

The unit is committed to achieving impact from its research. Its multidimensional strategy aims to implement the unit's achievements at the cutting edge of Applied Mathematics in research-intensive areas of engineering, science and medicine. In consequence, our industrial partners are those with powerful R&D departments. In accordance with Keele's ethos, priority is also given to addressing societal needs (e.g. in the areas of Primary Care and High School Education). In selecting impact areas, financial gains, although important, are not the primary consideration; instead the intrinsic logic of mathematical and technological development, and contribution to addressing environmental and societal challenges, are the priority.

For example, a broader use of railway transport and lightweight cars would be a major step towards a greener low carbon world. The perturbation approach developed at Keele has been employed by *Amsted* to improve safety and efficiency of freight trains. The unit will continue the strategy of collaborating with large multinational companies, for which impact can be facilitated by company-supported PhD students and Post Docs at Keele, as in the case of *Amsted.* Another important route for amplifying impact is through active participation in major EU networks, as was achieved in the collaboration with *TPV* in designing greener lightweight cars within the *EVA4GREEN* and *EAGLE* programs. These topics form the basis of the unit's Impact Case Studies (ICS).

Driven by Keele's regional agenda, the unit also plans to build upon the success of its novel approach to high-school mathematical education implemented by DB in deprived areas of Stoke-on-Trent. The outcome was a 40% increase in student participation in Post-16 mathematics, improvements in quality of the teacher workforce, and social mobility of students. Other collaborative activities include helping a local company, *Wade Ceramics,* to reduce their crack rate through improved mathematical modelling, which has potential for substantial impact in the longer term.

Impact related activities are embedded in research plans, recognised in workload allocation and through performance assessment & promotion. They also receive targeted funding where required (e.g. impact acceleration funding, from University and Faculty sources).

Research Integrity & Open Science.

Our Unit is at the forefront of developing an open research environment, and research outputs of all types are made openly available, which began in advance of the current REF policy. The Unit works with partners to seek external funding for open research, and where this has not been possible, utilises Keele's research repository. In accordance with the principles of the Concordat for Research Integrity the Unit strongly encourages the uptake of open research practices: for example, a quarter of staff deposit preprints at https://arxiv.org/, and consolidated codes and data complementing computation-intensive contributions are routinely shared via open-access platforms such as bitbucket.org, github.com and zenodo.org (e.g. Lucas: https://bitbucket.org/dan_lucas/psgpu/src/master/). In order to support a research environment which has a culture of integrity and best practice, research mentors work with ECRs to instil open science practices, regular workshops on open science are held for new staff and PGRs, and an updated summary of topics covered is distributed to all staff to enhance awareness of open science issues. To maintain and encourage the most rigorous research, all grant proposals undergo internal peer-review processes and successful proposals are deposited in a shared drive to facilitate best practice.

2. People

Staffing Strategy

The unit consists of 13 academics: 5 Professors (CJC (0.25 FTE), JJH, YF, JK, VS), 3 Senior Lecturers (SN, DP, PT) and 5 Lecturers (JC, DL, MJN, LP, PF), organised into 3 groups and 6 themes. In line with a new emphasis on data science, two lecturers, P. Ledger (PL) and P. Wootton (PW), have been appointed after the census date. During the assessment period the unit also hosted 5 Postdoctoral Research Fellows.

The unit's staffing strategy, embedded within the University's People Strategy, is guided by principles of equality, diversity and transparency, and by the obligations of the *Concordat to Support Career Development of Researchers*. Other key considerations informing the unit's staffing strategy are:

- Enhancement of areas of established research strength.
- Development of new research areas, while preserving cohesion.
- Support for interdisciplinary, collaborative research aiming to increase industrial and societal impact.

On this basis four excellent appointments have been made:

- DL brought in unique expertise in *Computational Fluid Mechanics,* which complements the *Fluids* group's traditional strengths and enables it to open new research fields.
- The appointments of MJN and LP reflect the FNS focus on materials research and greatly strengthen the whole range of activities of the Solids group.
- The appointment of JC aims at developing statistics research and establishing new industrial links and international multidisciplinary collaborations.

These appointments demonstrate the University's commitment to advance world-leading mathematical research. Recruitment achieved 50/50 gender balance. This commitment has been further emphasized by two after-census-date appointments in the area of *Data Science*. In particular, PL has brought unique expertise in *Computational Solid Mechanics* that will facilitate the Unit's expansion into data-driven computational mechanics.

The promotions of DP and SN to senior lectureships are of particular significance for the long-term sustainability of the Unit, fostering the next generation of academic leaders; SN also benefited from the *University Research Coaching Programme*. PT, initially appointed as a teaching fellow, on the strength of his achievements in algebraic number theory has been promoted first to a lectureship, and then to a senior lectureship.

Support for Early Career Researchers (ECR)

The unit is committed to support of ECRs. All ECRs are mentored by senior colleagues and attend a structured staff development programme, including mandatory training before undertaking PGR supervision. They enjoy reduced teaching and administrative duties and during their first and second year, they usually teach one and two modules a year, respectively.

ECRs can also participate in PhD supervision from the date of appointment, supported by experienced second supervisors. Two recent PhD scholarships for newly appointed lecturers were funded internally, and two relied on external support. The vibrant environment within the unit supports ECRs to become engaged in collaborative and interdisciplinary research. During the probation period their progress is regularly reviewed at specially organised meetings chaired by the Head of School. ECRs are expected to submit a first grant application to UKRI within two years of appointment; in doing so, they enjoy support and advice from their academic mentor and other Unit members. DL is a recent recipient of such an EPSRC award.

Support for all Researchers

Keele adheres to the *Concordat to Support Career Development of Researchers* and has held *HR Excellence in Research* award status since 2013. Staff within the unit are able to access a range of training and development opportunities offered to researchers across the university, including leadership training and coaching as detailed in REF5a.

The University *Equality and Diversity Committee* oversees EDI, through the University Research and PGR Committees which have this agenda within their remit. Staff involved in recruitment and assessment undergo mandatory training in unconscious bias and EDI.

Additional support for research-related activities (conference participation, invitation of overseas visitors, etc.) comes from the *Mathematics research account,* annually allocated from the School budget. Pilot grants (e.g. for preparing more substantial bids) are also available from the *Faculty Research & Development Fund* (FRDF). This support is particularly important for newly appointed staff (LP and JC have been awarded such grants in the first year of their appointment).

Development of researchers is Keele's strategic aim at all levels, with formal support and monitoring through *Annual Appraisal* procedures, tailored appropriately for the professorial staff (PPRE) and other academics (SPRE). This includes development of a research plan for those with a significant responsibility for research.

The *Work-Load Allocation Policy* within the unit aims at promoting world-leading research, based on publication quality, grant income, impact, and Post Doc and PhD supervision. Most researchers have one teaching-free semester annually, which is a core element of the *Unit Policy for Research Leave & Sabbaticals*. This not only allows extra time for concentrating on publications and grant proposals, and participation in scientific meetings, but crucially also enables staff to accept short-term visiting positions overseas without formal sabbatical arrangements at a higher level (MJN had a two-year secondment as a Marie Curie Fellow at the University of Cagliari, while YF, JK and VS have held multiple visiting Professorships in China, France, Italy, Israel and Turkey).

Keele actively encourages industrial collaborations. FRDF support has been crucial for the Unit in forging links with industry. For example, a joint effort with the US company *Ilrima,* focused on lubrication modelling, was initially funded through this source. The development of ICS with *Amsted* and *TPV* was also supported. The unit's key collaborators from *Amsted*, and *Tafe Motors and Tractors Limited* (India) are *Honorary Research Fellows* at Keele, while JK was seconded to *TPV* for six months during 2016-2019. CJC also worked with Thales on underwater noise modelling for the UK's nuclear submarine programme.

To facilitate links with local industry, *KRISP* (*Keele Research and Innovation Support Programme)* has been designed to support local SMEs. This programme is funded through the *European Regional Development Fund* (*ERDF*). The unit has participated in this programme: for example, CJC led a successful collaboration with the SME *EMS*, enabling the company to produce more efficient heavy-duty pumps.

The School actively supports the *wellbeing of staff and research students*. Staff or PGRs with special circumstances are given generous leave periods, and those with special medical needs are offered tailored arrangements (e.g. flexible work, extensive medical leave).

Support, Training and Supervision of PGRs

PGR students are recruited both nationally and internationally, and supported from diverse funding sources, including EU, the Commonwealth, and directly by industry. Since 2014 there have been 20 registered students (compared to 9 in REF2014 (+122%), all of whom completed successfully within 4 years. They have produced high-quality research, resulting in publications in high profile journals such as JFM, Proc. Roy. Soc., etc.

In keeping with Keele's PGR Code of Practice, an excellent supervisory and training environment is maintained. All PGRs develop a training plan as part of their Personal Development and Learning Plan. Supervisors are approved by the *Faculty PGR Committee*, with mandatory training provided by the Keele Doctoral Academy. Supervisory teams comprise at least two supervisors. The supervision style in the unit is "open door policy". All students enjoy modern research facilities and infrastructure, including access to the School's GPU cluster, private cloud, and licensed software.

The quality of PhD training at the Unit is also evidenced by the impressive destinations of PGRs. Former PGRs have secured postdoctoral positions at Warwick and Essex, and lectureships at Hull and Glasgow, as well as high-level jobs at *EDTF* and *GHD* London Offices. The 2019 *National Postgraduate Research Experience Survey* elicited a 100% response rate, with 100% for `*Overall Experience and Satisfaction'* (vs 82% for the sector average), 100% satisfaction for `*Supervision*', and 92% satisfaction for `*Resources*'.

Progress and welfare are monitored in accordance with University guidelines overseen by the *School and Faculty PGR Director*. During 2020, Keele established a Doctoral Academy which aims to provide a unified platform for PGR support, governance, and training. The university also has a thriving Postgraduate Association that specialises in representing the interests of postgraduate students. An elected PGR representative liaises with the PGR community.

Numerous regular and *ad hoc* research seminars broaden students' horizons. A well-established element of the PGR training is the *MAGIC* consortium of 21 UK universities, offering video conferencing access to over fifty advanced taught courses each year.

A generic training programme addresses the national agenda for key and transferable skills. In addition to obligatory internal presentations, all PhD students are strongly encouraged (and funded) to make regular conference presentations and attend advanced courses, in the UK and overseas. These have included various specialised courses at CISM, Udine, and Summer Schools in Cargese (Corsica) and Prague, where invited lectures/lecture series were delivered by the Unit's staff.

Training of PGR students has also been enhanced by collaborative activities within the framework of the ERASMUS+ programme. The unit has greatly benefited from PGR student and staff exchanges, with 16 universities from 9 countries supported by around £1M of ERASMUS+ funding between 2015-2020. The unit has hosted around 30 PGR students and a PGR international workshop with circa 40 participants. There have also been over 50 academic staff visits. This ongoing cooperation is a significant component of a truly vibrant environment for the Unit's PhD students. They not only enjoy interactions with their overseas counterparts and attend lectures by outstanding visiting scholars (e.g. Professors D.Natroshvili (Tbilisi), G.Mikhasev (Minsk) and V.Mykhaskiv (Lviv)), but also have opportunities for informal discussions. To the benefit of PhD students, several short, advanced courses have been supported by the School (e.g. *Applied Tensor Analysis* by Prof M.Itskov (RWTH, Aachen*), Structural Vibrations* by Prof I.Elishakoff (Florida Atlantic University, Boca Raton), and *Spectral Theory* by Dr A.Kiselev (St Petersburg)).

EDI

EDI is at the heart of Keele's culture. The university currently holds an institutional Athena Swan Bronze award, and the School was awarded the Athena Swan Bronze award in 2016 and 2020. The unit has sought to improve its gender balance through an action plan to improve female representation, which includes embedding of positive action statements in recruitment material. The unit's REF submission includes two female staff, which amounts to 17%. Over the REF period, the average percentage of female PGRs is above 30%. The unit also has 25% BAME staff, and a substantial number of BAME PGRs, as well as two professors above the standard retirement age.

The unit has a strong record of supporting flexible working. Over the REF period, instances of parental leave have been properly supported. Most staff have informal flexibility arrangements to handle their caring responsibilities. A convention of holding meetings between 10am and 4pm is followed. Minutes are circulated within a week of a meeting, to the advantage of staff not able to attend. The University maintains a fund for supporting return-to-research, from which staff within the unit have benefited.

Strict adherence to EDI principles in handling applications for promotions and internal funding, as well as workload allocation and appraisal, is ensured by the Head of School together with the unit lead. Formal appraisal activities are supplemented by intermediate research reviews for each staff member.

ECRs have individually tailored, protected allocations to address their training and research development needs, beyond the research and scholarship allocation of 30% for all research-active staff.

To promote diversity further, the School regularly reviews its EDI policies with the help of HR and the *School EDI Committee*. All School meetings are prefaced with an EDI reminder and the School Athena Swan/EDI Chair attends all School and Unit level events.

In addition to special welcome of applicants with protected characteristics in recruitment, recent positive action includes provision for anonymous staff/PGR feedback and introduction of guidelines for the support of any staff member, student, or visitor whose circumstances require privacy and/or special care.

All staff involved in REF leadership, which includes BAME professors, have received EDI and unconscious-bias training from the University. The institution's code of practice for REF has been followed and all deliverables have been thoroughly reviewed with acute attention given to EDI issues.

3. Income, infrastructure and facilities

Institutional framework

Keele research activities are supported by the *Directorate of Research, Innovation and Engagement (RIE),* which manages the *Research and Innovation Support Enhancement (RaISE)* team. RaISE works with academic staff within the Unit to identify and develop applications for research and infrastructure funding:

- Interdisciplinary collaborations are encouraged and facilitated by identifying optimal partners for leveraging Keele's infrastructure; crucially, incentives for joining forces are identified and flagged, with support from sources such as the Institute of Liberal Arts & Sciences.
- Industrial collaborations are supported by establishing local industrial links (e.g. *Wade*, *EMS)* and by providing seed-corn funding.
- International networks are promoted, with comprehensive support provided by the *Global Opportunity Team* at the *International Office* and the International Development Officer (RIE*)* (e.g. Marie Curie ITN *TANGO, ERASMUS+, etc.*).
- Resources are focused on key themes, in accordance with the priorities of *Sustainability and Health*, including the *Materials* theme promoted by FNS.

- A healthy variety of pilot projects, viewed as stepping stones for major grants, is funded on a competitive basis, primarily through the FNS vehicle *FRDF.*

The Unit is also supported through QR income, the distribution of which is devolved to Faculty and School level. Keele has also invested significantly in facilitating research impact. This resource is allocated on a competitive basis and focused on the potential to advance *Keele's Knowledge Exchange Strategy*. The unit is a clear beneficiary of this policy.

Unit's strategy for generating research income

Most staff activities aimed at generating external income and developing impact are coordinated by the *Mathematics Research Lead,* reporting to the *Head of School* and *Dean for Research*. The Unit's strategy is based on the following key elements:

- maximising the exceptional strengths of the two groups specialising in continuum mechanics,
- strong promotion of interdisciplinary research,
- industrial collaboration with R&D-intensive entities, primarily in the areas of engineering and medicine,
- development of major international networks involving world-leading scientists.

This strategy exploits a range of funding opportunities, including UK and overseas research councils, EU programmes, and industrial partnerships. Implementation of this strategy results in securing funding through responsive mode grants, substantial international networks, and fellowships, as well as small LMS conference and research project awards.

An important element of the unit strategy is the focus on bringing in outstanding PostDoc Fellows and building the foundations for the follow-up funded research. For example, three consecutive NERC grants awarded to VS funded PostDoc Fellows and prepared the ground for a substantial, successful research funding application. Other PostDoc Fellows have been supported by EPSRC (DL), EU (GAR), SERB-Royal Society (JK), *Amsted* (JK, GAR) and *TUBITAK* (DP). Continuous support from *ERASMUS+* brought in follow-up funding from the *Kazakh Ministry of Education and Science* and *RNF*.

Early Career Researchers (ECRs)

A fundamental aspect of the unit's long-term strategy is a multi-layered framework aimed at enabling ECRs to grow and secure their own research funding. Reduced teaching allocation for ECRs is a key component of this framework.

Within the unit, mentoring is taken very seriously, and through a constant dialogue with the mentor the individual's research agenda is developed. Specialised workshops with in-house and invited experts provide help in coaching younger researchers on writing grant proposals. Early drafts of proposals are thoroughly discussed with mentors and other staff members.

The effectiveness of this support infrastructure is evidenced by successes including a recent `*New Investigator*' grant (DL). Support for researchers in grant income generation is not confined to ECRs, and a similar collaborative approach is adopted for staff at the next stage of their careers. In particular, the awards obtained by MJN and DP also confirm the benefits of the stimulating environment.

EU Opportunities

Regular formal and informal sessions with UKRO (UK Research Office), lectures on forthcoming initiatives, and on anticipated changes in programmes and policies, provide Keele academics with better orientation in the changing environment. At all stages of grant preparation, contract negotiation, and reporting, the dedicated help of RaISE is crucial. At unit level, the strength of this user-friendly infrastructure is demonstrated by a large number of EU grants (MH, MJN, GAR, VS).

Research income

A well thought-through agenda of income generation has resulted in a high success rate in grant applications. Overall unit grant income has increased during the period, although the submitted

data shows income comparable to that of the 2014 REF. This is because considerable funding is not captured in the REF data (in particular, approximately £1M through Erasmus+).

Major awards

- £365K, 2019-2022, NERC, *Modelling wind waves. What lies beyond the significant wave height?* (VS).
- £351K, 2014-2018, NERC, *Towards modelling height probability distributions of "normal" and "freak" wind waves from first principles* (VS).
- £300K, 2011-2014, NERC, *New kinetic equations and their modelling for wind- wave forecasting* (VS).
- £209K, 2019-2021, EPSRC New Investigator Award, *Stabilisation of exact coherent structures in fluid turbulence* (DL).
- $980K, 2017-2021, US NSF-NERC, *Toward a new picture of the multifaceted meteotsunami* (VS) (Keele's proportion £63K).
- €129K, 2014-2018, EU FP7, *EU network air-sea interaction under stormy and hurricane conditions: physical models and applications to remote sensin*g (VS), (Keele's proportion ~ £14K)
- £1M, 2015-2022, *ERASMUS*+ ICM KA107 (DP).
- £126K, 2015-2017, Marie Curie, *Multi-scale modelling of waves in porous media with applications to acoustic control and biomechanics* (GAR).
- £91K, 2019, Royal Society-SERB, *Newton International Fellowship* (JK).
- £89K, 2014-2016, funding from *Amsted* to model longitudinal forces in freight cars (JK and GAR).

Other funding

As noted above, the unit has received substantial funding from other sources for research and engagement activities. MH during her affiliation with the unit within the assessment period (2014-2016) coordinated a major European Marie Curie ITN *TANGO*, *Thermo-acoustic and aero-acoustic nonlinearities in green combustors with orifice structures,* with total funding of €3.73M.

The unit has also a visible income stream (totalling about ~£500K) from the fees of 12 overseas PhD students, including a highly competitive Commonwealth Scholarship. An example of a small, but still meaningful, grant is the LMS award to LP for supervising a research project involving a disabled student.

Unit members have initiated several national and international projects, including the following examples. VS secured funding through the EPSRC *Maths Foresees Network* for the pilot project *Experimental modelling of wave dynamics on jet currents,* carried out at Hull in 2015-2017, jointly with S.Lukaschuk. Since 2018, JK has led a three-year basic research grant *Multiparametric dynamic modelling of layered strongly inhomogeneous elastic structures* funded by ARRS (Slovenian Research Agency). A major prestigious RNF (Russian National Foundation) grant, *Analytical models of seismic metamaterials*, was won in 2020 by JK (PI) and DP (CI). JK, DP and LP also participated in the grant by the Kazakh Ministry of Education & Science, *Development of asymptotic models for surface waves in solids with advanced mechanical properties, taking into account effects of coatings and embedded sources* (2017-2020). YF has had two grants awarded by NSFC (National Natural Science Foundation of China) that funded his collaborations with Tianjin University: *Analysis and experiments on localized bulging in inflated hyperelastic membrane tubes* (2014-2017, PI), and *Analytical and numerical methods for the post buckling and secondary bifurcation of film/substrate structures* (2017-2020, CI). The above grants, although not contributing directly to REF income, have added substantially to the long-term health of the unit. They have broadened the scope of funding opportunities, led to strategic international collaborations and high-quality publications, and further advanced the unit's research activities.

The robustness of the unit's strategy in grant income generation aimed at multiple sources is further evidenced by the even greater momentum after the REF census date. In particular, post-REF income has already reached 1.3 million over a six-month period, and includes:

- €1.82M (Keele's share €166K) EU, *Effective factorisation techniques for matrix-functions: Developing theory, numerical methods and impactful applications* (MJN).
- ~£100K, *CONACyT* (Mexico), *Ciencia de Frontera* programme, *La evolución de zonas activas entre placas tectónicas: un enfoquematemático basado en las vibraciones flexurales de baja frecuencia* (JK, DP).
- ~£100K, Kazakh Ministry of Education & Science, *Development of mathematical models and methods for analysing mechanical deformable systems with complex nonlinear properties and variable structure in machine dynamics* (JK, DP).
- ~£68K, NSFC, *Theoretical and experimental studies of necking of dielectric membranes subject to mechanical and electric loading* (YF).
- ~£321K, EPSRC*, Reducing the threat to public Safety: Improved metallic object characterisation, location and detection.* Project partners: DSTL, Rapiscan Systems, *Safeline.*https://magpoltensor.wordpress.com (PL).
- ~342K, EPSRC*, Generalised Magnetic Polarizability Tensors: Invariants and Symmetry Groups* (PL).
- ~£423K, EPSRC*, Object Detection, Location and Identification at Radio Frequencies in the Near Field.* (PL).
- ~£120K, EPSRC CASE Award Studentship (4 year PhD) with Siemens Healthineers, *A new in-factory machine learning surrogate model for the prediction of magnet quenching during manufacturing of MRI scanners (Siemens Contribution* ~£29) (PL)

Research impact

The unit's strategy prioritises academic, environmental and societal considerations in selecting sources of grant income, which has resulted in significant scientific, industrial and other impacts. Regarding scientific impact, this has led to top-quality journal publications over a broad area of Applied Mathematics. The best examples of industrial impacts with environmental implications have come from collaborative research in the areas of railway and automotive transport funded by grants from *Amsted* and EU, as detailed in the two ICS.

These efforts aimed at green transport modelling have resulted not only in industrial and ecological impact, but also in new mathematics, such as an original `almost rigid body' concept (JK et al., Mech. Time-depend. Mater. 2015), and a new multi-parametric approach for lightweight high-contrast laminates (LP et al., Math. Mech. Solids 2018).

The collaboration with Kazakh scientists supported by ERASMUS+ and its follow-ups resulted in a general dynamic analysis of drilling strings within the framework of structural mechanics (JK et al., Int. J. Eng. Sci. 2020), with the potential for efficiency improvements in oil and gas technology.

The TANGO Network generated, inter alia, new insight into the nature of thermo-acoustic phenomena in combustors, based on new exact solutions for the so-called intrinsic modes, and the discovery of a new class of instabilities caused by interaction of intrinsic and acoustic modes (VS & N.Mukherjee, Comb. & Flame 2017, Comb. Sci. Tech. 2019).

Organizational support and facilities

The unit continues to benefit from University strategic investment in research excellence, reflected in sharply increased staffing levels. Support mechanisms include direct subvention to the School budget based on REF 2014 performance, resulting in reduced teaching loads and increased research time. All unit members hold *Personal Research Accounts* which, over and above other internal support, are used to promote individual long-term research activities and add to academic independence.

RaISE provides specialist technical assistance; it facilitates awareness of funding opportunities, ensures support for grant applications, including costing/pricing and overhead recovery

(sometimes at short notice), is indispensable in contract negotiations, and is instrumental in patenting and licensing.

Computer facilities

The unit is part of the School of Computer Science and Mathematics, which ensures access to significantly better high performance computer facilities and technical support than most other mathematics departments of similar size. This excellent provision enables our world-leading research in continuum mechanics and supports recent developments in statistics and data science. The School maintains an HPC cluster comprising 512 virtual CPU cores and GPU capacity of 256 teraflops, serviced by a School-based team of dedicated technicians, as well as a Private VScaler cloud. The facility is open to all Unit staff and PGRs.

More advanced computational resources are also available when needed; for example, VS has a long-standing arrangement through the Met. Office to access one of the world's largest supercomputers at the *European Centre for Mid-Range Weather Forecasting.*

DL has recently been awarded by *Nvidia* two *Titan XP* GPUs. These were crucial in the preparation of his successful 2019 EPSRC proposal, which funded the purchase of a high-performance workstation consisting of a 10 core Intel Xeon W-2255 and a 5120 CUDA core Nvidia Quadro GV100 32GB GPU.

The unit's current access to superb computational facilities, including those outside Keele, is invaluable for the unique direct numerical simulations of nonlinear wind wave long-time evolution carried out by VS and S.Annenkov, and crucial insight into emergence of turbulent patterns by DL.

Members of the unit are active in *cross-HEI collaborative use of major research facilities* (large air-sea interaction tanks) at Heidelberg University, Hull University, Marseille University, N.Novgorod Institute for Applied Physics and Tel-Aviv University.

In-kind contributions include access to vast databases of wave observations in the ocean (Scripps Institute, US; Melbourne University, Australia; CNRS, France), as well as various contributions from *WADE* and NHS institutions.

4. Collaboration and contribution to the research base, economy and society

Contribution to the scientific community and health of the discipline

Through Keele's flexible approach and strategic support for collaborative activities, the unit continues to provide a significant contribution to the UK and international scientific community by willingly providing its expertise:

- as members of various national/international panels (e.g. IMA Lighthill-Thwaites prize (YF), Roy. Soc. London (GAR), MIUR, Italy (JK), ARRS Slovenia (JK));
- in assisting research funding agencies in the UK (EPSRC, NERC, Leverhulme) and worldwide (e.g. China, Canada, Georgia, Germany, Ireland, Israel, Norway, Russia, US) by regularly reviewing grant applications and sitting on grant panels;
- in PhD and DSs examinations and procedures (UK (CJC, YF, JJH, JK, SN, MJN, DP, GAR and VS), Australia (CJC, VS), China (YF), Denmark (JK), France (JK, VS), India (JK), Ireland (VS), Israel (VS), Italy (MJN), Norway(VS), Pakistan (JK), Turkey (JK)) and promotion panels and procedures (China, India, Israel, Pakistan, Slovenia and US).

The contribution of Keele Mathematicians to the future health of the discipline goes well beyond its core area of expertise. The unit supports the basic infrastructure of the discipline by being good citizens in many invisible ways, including refereeing of papers and research proposals, and extensive membership of committees. The unit's contribution to the advancement of continuum mechanics is enhanced by the exceptional coherence of the *Fluids* and *Solids* groups: many of their activities (including conference organisation and industrial collaborations) have been made possible by the intensity and diversity of the intergroup collaborations. Events such as the

traditional *UK Elasticity Days*, where Keele is a key player, further illustrate the role of the Mathematics Unit as a team.

Key editorships

- *IMA Journal of Applied Mathematics* (YF (till 2018) and JK)
- *ZAMP* (JJH)
- *Nonlinear Processes in Geophysics* (VS)
- *Journal of Sound and Vibration* (CJC)
- *Mathematics and Mechanics of Solids* (JK)
- *Mechanics of Time-Dependent Materials* (JK)
- *PMM Journal of Applied Mathematics and Mechanics* (JK)
- *Journal of Mechanics of Materials and Structures* (YF)
- *Mechanics of Soft Materials* (YF)
- *Acta Mechanica Sinica* (YF)
- *Journal of Engineering Mathematics* (YF)

Other editorial activities include: editorship of the Springer book Series on Nonlinear Science (VS) and of special volumes and journal issues (e.g. the Proceedings of the Royal Society A Special Issue in honour of Prof P.Chadwick, FRS, edited by YF and JK jointly with Prof R.W.Ogden, FRS).

Meeting organisation/Scientific Committees (examples)

Staff within the unit have contributed to organisation of a number of major scientific events, including:

- *2015* EUROMECH Colloquium No. 574 on *Recent Trends in Modeling of Moving Loads on Elastic Structures*, Anadolu University, Eskisehir. Turkey (JK, co-chairman).
- 2015 *International Workshop on Pattern Formation in Soft Materials*, Tianjin University, China (YF, co-chairman).
- 2015 12th *International Conference on Vibration Problems*, Scientific Committee and Steering Committee, IIT Guwahati, India (JK).
- 2015 *Elasticity Day*, Keele, UK (YF, organiser).
- 2015 *International Conference on Mathematical and Computer Modelling in Continuum Mechanics* (dedicated to 105th anniversary of V.V. Novozhilov), Organising Committee, Saint Petersburg (DP).
- 2017-2020 *European Geophysical Union General Assembly*, Vienna (VS, convener).
- 2019 *15th International Conference on Dynamical Systems – Theory and Applications* (DSTA 2019), Scientific Committee, Lodz (JK).
- 2019 *Workshop on Algebra*, Keele (PT, organiser).
- 2020 Online conference, *Hopf algebras and Galois module theory* (PT, co-organizer).
- In 2021 the Unit will also host EUROMECH Colloquium No. 626 *Mechanics of high-contrast composites* (DP, organizer).

Major presentations

All submitted staff have had numerous invited presentations at important national and international events.

- 2015 *4th International Conference on Topical Problems in Continuum Mechanics*, Tsakhkadzor, Armenia. *Keynote Lecture* (JK).
- 2016 *7th Int. Conf. Continuum mechanics and related problems of analysis.* (Dedicated to 125th birthday anniversary of N. I. Muskhelishvili), Batumi, Georgia. *Keynote Lecture* (DP).
- 2018 *Mathematics and Mechanics: Natural Philosophy in the 21st Century*, Oxford, *Plenary Lecture* (YF).
- 2019 *Isaac Newton Institute, Industry Day*. *Industrial Applications of Complex Analysis*. *Plenary lecture* (CJC).
- 2019 *Distinguished Lecture Series*, Florida Atlantic University, USA (JK).

- 2020 *International Conference on Advanced Problems in Mechanics* (APM-2020), Saint Petersburg. *Plenary lecture* (JK, DP).

Advanced training courses

Staff from the unit have delivered a range of advanced training courses for early career researchers, including:

- 2015 Programme for young scientists and PhD students *Rogue and Shock Waves in Nonlinear Dispersive Media*, Cargèse, Corsica, Institut d'Études Scientifiques (VS).
- 2017 International Workshop for postgraduate students in Applied Mathematics, Keele (DP, organiser).
- 2017 *Festival de Théorie 2017*: *Avalanching & Self-Organization in Plasmas*, Aix-en-Provence, France (VS).
- 2019 Summer school *Wave propagation in microstructured media*, Cargèse, Corsica, Institut d'Études Scientifiques (JK, MJN).
- 2019 INI Summer School *Bringing pure and applied analysis together via the Wiener–Hopf technique*, Cambridge (CJC, MJN).

Indicators of Recognition

- *Fellow of European Academy of Sciences* (JK, since 2019).
- *Sackler Fellowship* (Sackler Institute for Advanced Studies, Tel-Aviv University), 2016-2017 (VS).

Visiting Professorships

- Anadolu University (JK)
- Modena University (JK)
- Bretagne-Sud University (JK)
- Tanjin University (YF)
- Toulon University (VS)

Interdisciplinarity and contribution to economy and society

Institutional Support

RaISE provides the crucial support underpinning engagement and partnership activities. For example, industrial collaboration with *Amsted* and *TPV* was facilitated through an Impact Acceleration pilot grant. Collaborative efforts with the lubrication industry, structural engineering, and other activities with potentially substantial impact are also supported. The key users of the Unit's research are engineers, biomedical scientists and geoscientists (though the circle of beneficiaries is much wider). The focus is on the following main research areas contributing to economy and society.

Engineering

Collaboration with the engineering world is carried out through engagement at three structural levels:

- At *local level,* the Unit collaborates through *KRISP,* engaging staff, students and SMEs. For example, *KRISP* supported a project with *EMS*, a company which manufactures pumps for the water-waste and biogas industries. The challenge faced by the company was that the pumps they manufactured were not performing at the optimum energy efficiency demanded by different field conditions. Keele mathematicians, led by CJC, developed a robust mathematical model, which enabled the company to optimise the pumps to different environments, making them more efficient.
- At *national level,* the Unit has worked with *Thales UK.* CJC played a key role in the government-funded study of noise generation in the context of the `Successor Programme' for the next generation of nuclear submarines, which is fundamental to national defence.
- At *international level*, theoretical results obtained in the area of the dynamics of inhomogeneous solids are being used in the railway and automotive industries for developing modern trains and "green cars". In addition, progress in modelling of "green combustors" at Keele

and through the Keele-led ITN network resulted in safe and efficient designs of combustors emitting reduced amounts of undesirable substances (MH and VS).
- A recently established strategic Keele-Saint Petersburg network in the area of seismic metamaterials (supported by a substantial grant from RNF, JK-PI, DP-CI) is centred on earthquake protection of critical industrial infrastructure.
- A collaborative activity with a group of Slovenian Universities and research institutions on modelling of high-contrast laminates (supported by a major grant from ARRS, JK-PI, GAR-CI) has developed a theoretical framework for acoustically engineered structural design.
- Long-term collaboration with Al-Farabi Kazakh National University on surface wave modelling (supported by a three-year grant from the Kazakh Ministry of Education and Science) has resulted in findings with potential for seismic control (JK, DP and LP).

Biomedicine
- Mathematical modelling of aneurysm formation and rupture, jointly with Tianjin Medical University (YF).
- Dynamic cell patterning through controllable and variable surface wrinkles, jointly with the Institute of Science and Technology in Medicine (ISTM) at Keele (YF).
- Modelling of blood-clot removal after strokes, jointly with Royal Stoke Hospital and ISTM (SN).
- Modelling of bioreactors for growing of human tissues, jointly with ISTM and the Oswestry Hospital (SN).
- Statistical modelling in primary care supported by Arthritis Research UK, North Staffordshire Primary Care Consortium for NHS (JB).

Geosciences
- Reconstruction of palaeo-tsunamis based on their sediment footprint, jointly with Keele's applied geophysicists (VS).
- Modelling of lava flows with Keele volcanologists and Bristol School of Earth Sciences (SN).

Education & Society

The unit's priorities also include providing support to local communities through a variety of joint activities with local industry and high schools.
- A collaborative research programme by Keele mathematicians and psychologists led by DB has been implemented within the *Stoke-on-Trent Mathematics Excellence Partnership*. Novel approaches developed for teaching STEM subjects in deprived areas have resulted in substantially improved educational outcomes and a palpable increase in social mobility. The significance of these new pedagogical approaches, implemented regionally, is of national importance.
- Collaboration with the local ceramic company *Wade Ceramics* (YF, SN, together with Liverpool University) in a deprived area of Stoke-on-Trent aimed not only at improving existing technology by using advanced modelling, but also at addressing societal needs by developing research-based manufacturing.

Networks

Numerous national and international networks (ERASMUS+, INI, ICMS, EPSRC, EU, National Institute for Healthcare etc.) were instrumental in producing quality and breadth of research far exceeding what might be expected *a priori* from a unit of this size. Examples are:
- EPSRC *Maths Foresees* Network (2014-2018) (VS).
- A series of back-to-back three-year access arrangements to use the ECMWF (European Centre for Medium-Range Weather Forecasting) supercomputer (500 kunits/year) from the MetOfice/ECMWF to facilitate the existing collaboration with the ECMWF (VS).
- Regular participation in the *Woods Hole Geophysical Fluid Dynamics summer program* (DL).
- *Primary Care Research Network at the National Institute for Healthcare* (JB).
- A collaborative EU grant on air-sea interaction under hurricane conditions involving researchers from France, Germany, Finland. Russia and UK (VS, coordinator).
- Numerous EPSRC-funded SIGs (Special Interest Groups).

INI

Another boost to the Unit's networking is provided through strong interaction with the INI *(Isaac Newton Institute for Mathematical Sciences*, which includes the *Gateway and Knowledge Transfer Network* (KTN)), and ICMS (*International Centre for Mathematical Sciences*). The unit maintains regular contact with INI & ICMS via its Keele representative (VS). Unit members have been prominent in INI & ICMS events. For example, both CJC and VS have been fully-funded residential participants in INI programmes: VS in the programme *Water Waves* (July-August 2014), and CJC in the two programmes *Bringing Pure and Applied Analysis together via the Wiener-Hopf technique* (August 2019) and *Complex Analysis: Techniques, Applications and Computations* (September–December 2019); and VS participated in the 2016 ICMS programme *Trapped Waves*. In addition, CJC gave plenary talks at shorter INI meetings, including *Mathematics in the Spirit of Joe Keller* (2017), and a meeting dedicated to the work of I.D.Abrahams (2018). The unit also benefits from visits from prominent overseas scientists who participate in INI's programmes.

Collaborative arrangements for PGR training

- *MAGIC Maths:* a consortium of 21 research-intensive universities which share live video broadcasts, and assessment, of a wide range of advanced PGR courses.
- *ERASMUS+*: this major EU-funded network has supported Keele's collaboration with 16 universities from 9 countries. PGR-oriented lectures by eminent academics have been organised along with conferences for young scientists and individually tailored training.

Further support of research visits

Inbound travel. During the REF period the Unit hosted well over 100 visitors from all over the world. Most of these visits were funded through the aforementioned networks, in which the combination of the School Research account and staff individual research accounts enabled great flexibility.

Outbound travel. There is a healthy level of national and international travel funded primarily through networks and grants and supplemented by Faculty, School and individual research accounts.