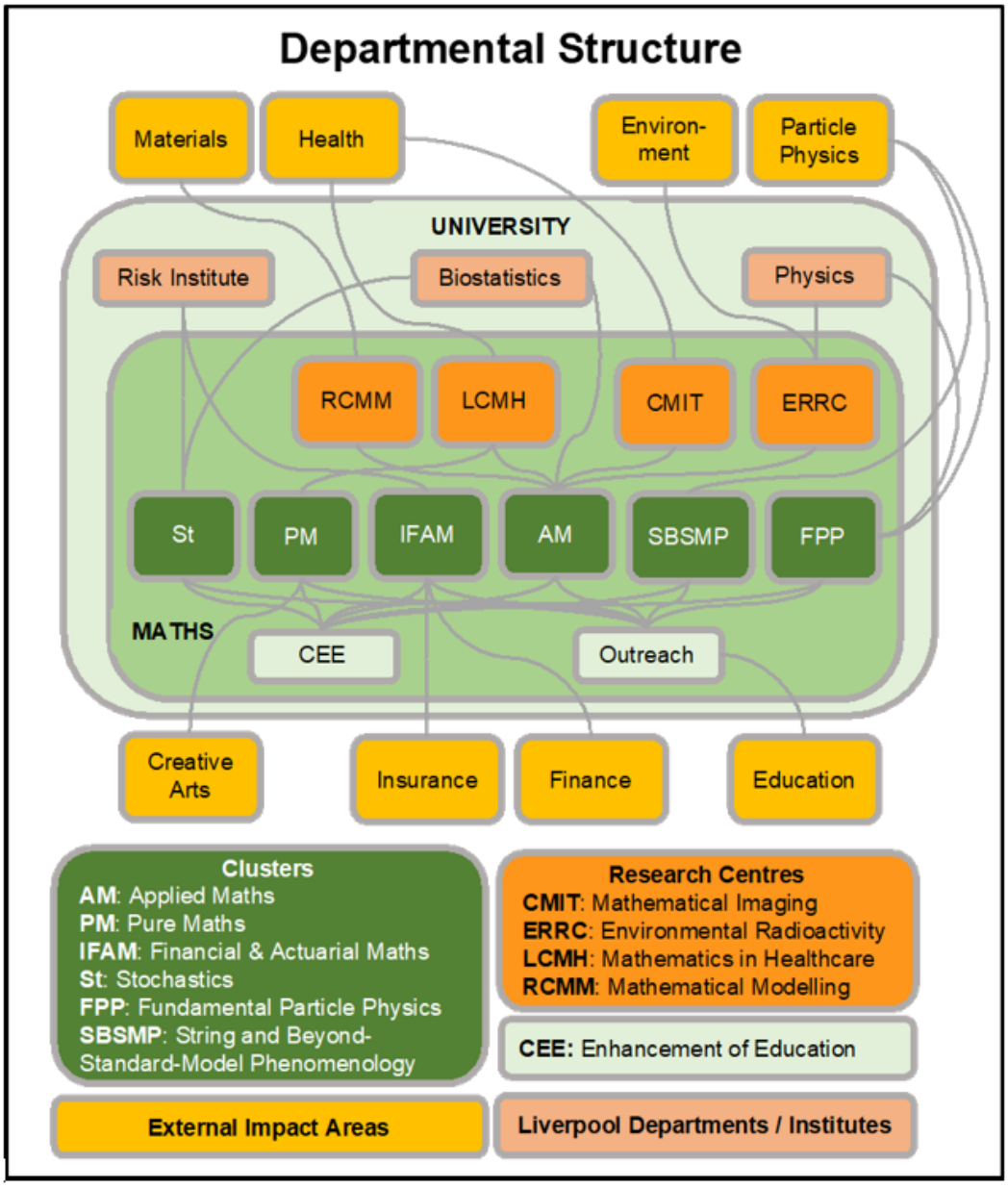
Institution: University of Liverpool

Unit of Assessment: UoA10

1. Unit context and structure, research and impact strategy

1.1 Unit Context and Structure

The *Department of Mathematical Sciences* is a substantial (53.38 Category A FTE) unit which together with Chemistry and Physics comprises the *School of Physical Sciences*. We are dedicated to exploration and discovery across the full spectrum of mathematical sciences, from its traditional core to its interdisciplinary interfaces with medicine, biology, materials science, finance and physics. We apply these discoveries and their spin-offs, use the methods developed for the wider benefit of society, and share the excitement of mathematics with a broad audience, inspiring future generations of mathematicians, particularly those from under-represented groups.



The department is structured into six research clusters which catalyse fruitful interactions between staff with related interests and concentrate critical mass in areas of excellence. These are overlaid by four intra and inter-disciplinary research centres which facilitate impact. The *Research Centre for Mathematical Modelling* (RCMM) provides internal seed funding and support for research visits and workshops. The *Liverpool Centre for Mathematics in Healthcare* (LCMH), the *Centre for Mathematical Imaging* (CMIT), and the *Environmental Radioactivity Research Centre* (ERRC) channel impact in health and environmental modelling.

Academic staff contributing to the research environment in census period:

	Pure Mathematics				
AG	Eckl	Guletskii	Mertens[E]	Pagani	Pukhlikov[P]
	Rizzardo[E]				
D	Hall	Rees[P] (em)	Rempe-Gillen[P]	Meyer	Nair
GT	Giblin[P] (em)	Goryunov[P]	Karpenkov	Pratoussevitch	Woolf
	Applied Mathematics				
WSMI	Alpers	Appleby[P] (em)	Chen[P]	Colquitt	Haslinger[E]
	A Movchan[P]	N Movchan[P]	Piliposyan	Selsil	Thompson
	Valkonen[E*]				
MB	Bearon[P]	Domijan[E]	Lewis	Sharkey	Vasiev
	Institute for Financial & Actuarial Mathematics				
	Assa	Azmoodeh[E]	Boado Penas	Constantinescu[P]	Deshpande[E*]
	J Eisenberg[E]	P Eisenberg[E*]	Gashi	Menoukeu-Pamen	Mitra[*]
	Pantelous[*]	Papaioannou	Patie	Rojas Nadayapa[E*]	Sahin
	Zhu[E]				
	Stochastics				
	Konstantopoulos[P]	Li[*]	Liu	Piunovskiy	Yuan[E]
	Zhang	Zychaluk			
	Fundamental Particle Physics				
	Buividovitch	Gorbahn	Gracey[P]	Jack[P]	Langfeld[P*]
	Rakow	Schaich[E]	Teubner[P]	Vogt[P]	
	String and beyond-Standard-Model Phenomenology				
	Faraggi[P]	Hardy[E]	Mohaupt	Parameswaran[E]	Tatar

AG=Algebraic Geometry	Female
D=Dynamics	ECR[E]
GT=Geometry and Topology	Professor[P]
WSMI=Waves, Solid Mechanics and Imaging	Left before census date[*]
MB=Mathematical Biology	REF period appointment

1.2 Research and impact strategy

1.2.1 Strategy: We have six strategic priorities:

S1 *Support and develop sustainable research clusters;*
S2 *Empower staff to conduct outstanding research and impact;*
S3 *Enhance collaborative and interdisciplinary activity;*
S4 *Connect to priority research themes of UK Research Councils;*
S5 *Build a research environment embodying the principles of Equality, Diversity and Inclusivity;*
S6 *Promote mathematics to the wider public and develop a new generation of mathematical researchers.*

The first four are analogues of our REF2014 research priorities but with impact embedded. This reflects our recognition of impact as core to our research and mission. During the REF period we have made excellent progress in engaging staff across the department in impact. New priorities (S5) and (S6) reflect our increasingly inclusive and outward-facing department and will help us cement these positive developments in the future.

1.2.2 Achievements: We illustrate the success of our strategy through some of our major achievements in this REF period.

Investment in people: The department has an ambitious, ongoing, programme of appointments, aimed at developing our research clusters (S1) and enhancing interdisciplinary activity (S3). We have increased academic staff numbers by approximately 30% since 2014 (§2) and have

(i) Initiated a new *Stochastics* cluster (S1);
(ii) Supported grants with appointments aligned with our strategic priorities (S3);
(iii) Established a leading research group in *Financial & Actuarial Mathematics* (S1);
(iv) Built up the existing *Algebraic Geometry* group into one of the largest in the UK (S1).

We hire outstanding individuals at all career stages: early career academics in our well-established research clusters to ensure long-term vibrancy and sustainability, and research leaders in the new *Stochastics* cluster to guide its formation (§2.1). We host major conferences to disseminate our ideas and bring together mathematicians from around the world to stimulate new research (S2).

In 2019 we appointed two (3 year, 50% FTE each) Impact Officers to form the Impact Group, together with Impact Coordinators Sharkey and Chen. The group supports individual staff, and the research centres, in developing impact internally and engages with industry and other third parties to promote our research externally (S2, S3, S4). The Impact Officers have led staff development sessions and embedded a new culture of impact amongst staff.

Liverpool Centre for Mathematics in Healthcare: In 2016 we established LCMH (S2) through an EPSRC large grant (PI: Chen, 2015-2020, £2.5M) and the appointment of three academic staff (Domijan, Colquitt, Valkonen). Eight academics from the pure and applied clusters are Co-Is, alongside 13 further Co-Is from the University (S3), and 19 external partners including Unilever, Astra-Zeneca and Liverpool Health Partners (S4). LCMH has generated >100 papers, over £7M of further investment to LCMH Co-Is, and had significant impact via

- Alder Hey Children's hospital and TRITEC Developments Ltd on infection control in hospitals;
- Public Health England modelling antimicrobial resistance in Shigella;
- Modelling of the Nellix endovascular aneurysm sealing system;
- Royal Liverpool University Hospital, Walton Neurological Centre, Image Analysis Group (London) and Mirada Medical (Oxford) developing clinical image-processing algorithms.

See §4.2.1.

Mathematical Foundations of Metamaterials: The Waves and Solid Mechanics Group maintained their strong international profile as leaders in the mathematics of metamaterials. They successfully bid for the EPSRC Programme Grant (EP/L024926/1, 2014-20, Co-Is: A Movchan, N Movchan, Thompson, Investigator funder contribution £489K) on *Mathematical Foundations of Metamaterials for Multi-Scale Physics and Mechanics* with Imperial College London and Liverpool John Moores University.

Pump-priming from the Research Centre in Mathematical Modelling helped the group build on two major collaborative EU grants (PARM-2, 2012-15 and INTERCER2, 2011-15) and was instrumental in securing two more (CERMAT2, 2014-17 and INDUCE-2-SAFETY, 2014-17) supporting PDRAs, workshops, visits and industrial collaborations. See §4.2.2.

Institute for Financial and Actuarial Mathematics: IFAM, formed in 2011, has established itself as a global leader for research in financial and actuarial mathematics, and for collaborative training of junior researchers. It is recognised as Number One in UK in Actuarial Research in the Non-Business Schools category, and number 5 in the world[1].

Eight appointments (S1) have been made since 2014, including Elise Richter Fellow J Eisenberg. Key successes include securing EU Framework project on Risk Analysis Ruin and Extremes (S3) and the four-year appointment of Menoukeu-Pamen as the Alexander von Humboldt Foundation's "German Research Chair in Mathematics and its Applications" at AIMS Ghana in 2016 (S2, S5, S6). See §4.2.4.

Core strength in Theoretical Physics: The importance of our research in theoretical physics, and strong links with the *Physics* department, is recognised though STFC consolidated grants from both Theory and Experimental Particle Physics panels. The *Theoretical Physics* cluster was reconfigured into *Fundamental Particle Physics* (FPP) and *String & beyond-Standard-Model Phenomenology* (SBSMP) to sharpen research foci, with two appointments in each to ensure their success (S1). We have strong collaborative networks with experimental groups (REF panel B9) locally and internationally, including CERN and Fermilab, USA (S3, see §4.1.2).

Public engagement and creative collaboration: The department has a proud history of successful outreach organised through our dedicated outreach team (2 FTE). Recognising this expertise, the Department for Education announced in July 2018 that Liverpool will be the first city in the North of England to host a specialist 16-19 mathematics school. The University of Liverpool Mathematics School, opened in September 2020, builds on the strengths of our current outreach and acts as a hub to engage more schools throughout the region (§4.4.2).

[1]Worldwide University Rankings based on research contributions from the top five leading actuarial and risk management & insurance journals.

In 2015 we hosted a Leverhulme-funded artist in residence (PI: Rempe-Gillen), Emily Howard. This led to staff involvement in the Royal Northern College of Music's centre for Practice and Research in Science and Music (§4.4.3).

1.2.3 Implementation during the REF Assessment Period: We outline the strategic initiatives which underpin the achievements in §1.2.2, and on which future achievements will be built.

Increase in academic staff: During the REF period we have invested substantially in staff to strengthen existing research areas and create new ones (S1) see §2; appointed 34 PDRAs associated with research grants (S1, S2, S3); and invested in 6 teaching and scholarship positions to develop innovative resources which free up academic time for research (S2) and contribute to impact though research-connected teaching and industrial links (S3).

Impact: We merged the previously separate research and impact strategies by embedding impact and creating the Impact Group (§3.2.3). The breadth of our REF impact case studies (health, environmental modelling, financial and actuarial applications, and public engagement) illustrates our success in inculcating a broad culture of impact.

ICS1 The COMET initiative establishes international standards for clinical trials policy, guidance and patient participation.
ICS2 Mathematical modelling of an aneurysm sealing system triggers patient safety policy that withdraws surgical practice from the NHS.
ICS3 Market first insurance product establishes start-up Stable Group Ltd to provide financial security to food and farming businesses.
ICS4 Pure mathematics informs music to wide critical acclaim and establishes new composer practice.
ICS5 Radiometric dating at the Environmental Radioactivity Research Centre changes environmental policy in Norway and Canada and redirects commercial resources.

Our approach encompasses consultancy (ICS3,5) and support for innovative collaborations (ICS4). Activity associated with four of the case studies has received EPSRC IAA or HEIF funding (ICS1-4). Our research centres play a pivotal role: ICS2, 3 and 5 arise respectively from the Liverpool Centre for Mathematics in Healthcare, the Institute for Financial and Actuarial Mathematics (IFAM) and the Environmental Radioactivity Research Centre (§4.2.3). ICS3 contributes to research and industry-connected teaching through the IFAM summer internship programme (§4.2.4). ICS3 and 5 originated in contracted consultancy; this is uncapped and staff can undertake projects privately or through the university. The author of ICS3 is Head of Consultancy for the department (S2). ICS2 and 5 are underpinned by interdisciplinary research with clinicians and environmental scientists (S3). ICS4 brings together science and the arts to communicate research to the public in original ways (S6).

Interdisciplinarity: We either initiated, or contribute to, the following interdisciplinary research centres within the University (S3):

(i) Liverpool Centre for Mathematics in Healthcare
(ii) Risk Institute
(iii) Centre for Mathematical Imaging Techniques

(iv) Environmental Radioactivity Research Centre.

Interdisciplinarity is an explicit consideration when seed funding proposals are evaluated by the Research Centre for Mathematical Modelling.

Development, equality and diversity: We initiated a successful support programme for staff fellowship applications (§2.1.1). We developed support for promotion with an emphasis on encouraging under-represented groups (§2.3). Female staff Bearon and Constantinescu exemplify the success of this strategy: both joined the department as ECRs, and since 2014 have been promoted to Reader and then Professor. Bearon, now HoD, was supported through periods of flexible working (0.6-0.8FTE, 2011-2017).

Governance: We stream-lined governance in 2016 by re-organising into Executive, Research, and Teaching Committees, operating transparently with agendas and minutes available on the staff intranet. Executive Committee develops and implements strategy, including appointments; Research Committee oversees research and impact policy, which is implemented by the Impact Group.

Good staff communication is ensured by the Annual Staff Meeting, the Staff Away-Day, and the monthly Departmental Bulletin informing staff of developments within the University and wider sector, as well as departmental policies, success stories, and events. Overall leadership is provided by the HoD, advised by the Executive Committee.

Open Research: Our policy is that our research is freely and openly available. Publications are deposited in the University's research repository, and their availability is confirmed as part of the annual REF reading programme. We also make extensive use of the *arXiv*. Gold open access publishing in the REF period has been supported by £49k of UKRI and institutional funding.

The University provides facilities for researchers to write data management plans at the application stage for external funding. It also provides the Liverpool Data Catalogue, a service for creating a record of data to facilitate open discovery, together with a data repository where data may be deposited and identified through a unique DOI. We use GitHub for open-source software development and release, and Zenodo for open data releases.

Research Integrity: The department has a trained ethics lead (Nair). All academic staff and researchers complete an on-line 'Research Ethics Training' module to ensure that research is conducted to the required ethical standards. This covers research with human participants, ethics and regulation, informed consent, confidentiality, liability and inducement. Researchers have also completed obligatory training courses on: EU General Data Protection Regulation (GDPR) and Information Security Essentials, the Bribery Act 2010 and Managing Safety at the University. PGRs receive Ethics training through the Liverpool Doctoral College core modules in Ethics and Governance. Research Ethics is embedded in PGR Induction.

Ethical approval is required for all research and outreach undertaken by staff or PGRs that involves human participants. The School and/or Departmental ethics leads advise on how to submit applications.

1.3 Future strategy and plans

The priorities S1-S6 will guide our strategy in the next five years.

1.3.1 People: Sustained increase in UG student numbers allows us to continue to invest in academic staff to maintain and strengthen our areas of excellence and seize new opportunities. We will develop our six research clusters in ways that foster collaboration within the department (S1). Consonant with our strategy, three new ECR appointments (August 2020) have been made in *Theoretical Physics* and *Stochastics*; further positions in financial and actuarial mathematics are in recruitment; one position in Pure (*Dynamics*) is on hold due to Covid-19.

Building on our investment in *Stochastics* and the successes of LCMH and IFAM we will lead a CDT bid in the next funding call connecting fundamental mathematical research to applications in health and finance (S3, S4).

The School of Physical Sciences attained an Athena SWAN Silver award in November 2016 (§2.3). We are addressing gender equality across all career stages, from school outreach activity through to senior leadership and hope to submit for Athena SWAN Gold in the next REF cycle. The School ED&I group are developing local actions to support the University's commitment to signing the Advance HE Race Equality Charter.

1.3.2 Support structures: We restructured the Research Committee in 2019/20 to create new academic leads for Research Support and Environment tasked with providing tailored local support for grant writing and the organisation of seminars and workshops.

The support lead informs staff of relevant grant opportunities and UKRI priority areas (S4). This strategy has led to successes such as the STFC grant *Quantum Sensors for the Hidden Sector* (UoL lead Hardy, £4.8M, October 2020). We aim to lead, or contribute as Co-I, to at least one major interdisciplinary bid annually.

The Environment lead has supported the transition to a virtual seminar culture in response to Covid-19 restrictions. Cluster organisers have grasped this opportunity to form joint seminars with other institutions (IFAM with Leeds, Waves Group with Keele, Mathematical Biology with LJMU and Manchester) and to invite speakers from outside the UK and Europe. We will retain these benefits when in-person seminars become possible again (S2). The Terry Wall lecture (Prof. Claire Voisin, College de France) and Barkla lecture will run virtually in 2020-21.

1.3.3 Interdisciplinarity and Impact: We will utilise our influence through professional bodies, especially the Institute and Faculty of Actuaries and the Institute of Mathematics and its Applications (Bearon on Council since 2017) to champion under-recognised areas on the interface of mathematical sciences with other disciplines, for example Actuarial Sciences, to UKRI (S3).

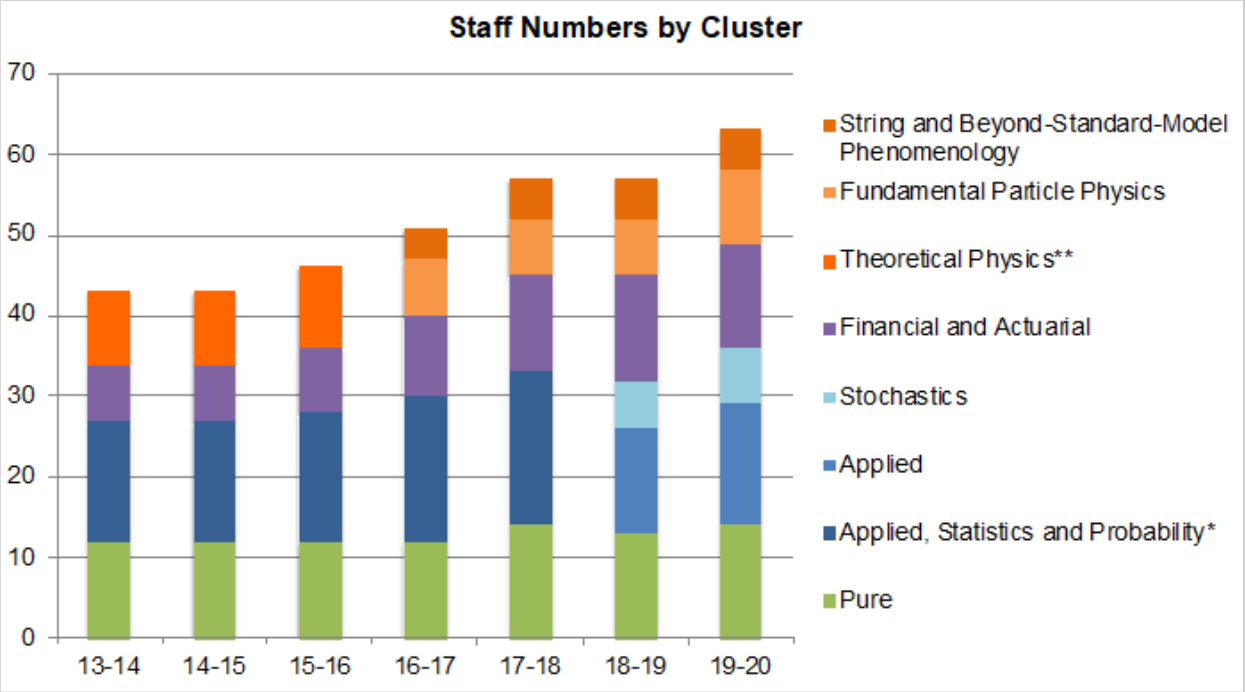
The Impact Group will sustain and develop our links with industry by hosting problem-solving events with industrial partners and conducting individual impact development meetings with staff across all clusters to inform the next impact plan. We will develop a communications strategy with the newly appointed Science Communications Manager in the School of Physical Sciences to enhance our external profile, and further facilitate impact (S3, S6).

1.3.4 Public engagement: Our ethos of public engagement, now formalised as priority S6, has enabled our outstanding outreach programme, Leverhulme artist-in-residence, and involvement with the African Institute of Mathematical Sciences. We will sustain this range of activity as well as seeking new ways of communicating mathematics and inspiring new mathematicians. We anticipate enhanced outreach possibilities associated to the UoL *Mathematics School* and will incentivise staff involvement by formally recognising time committed.

2. People

2.1 Staffing strategy and development

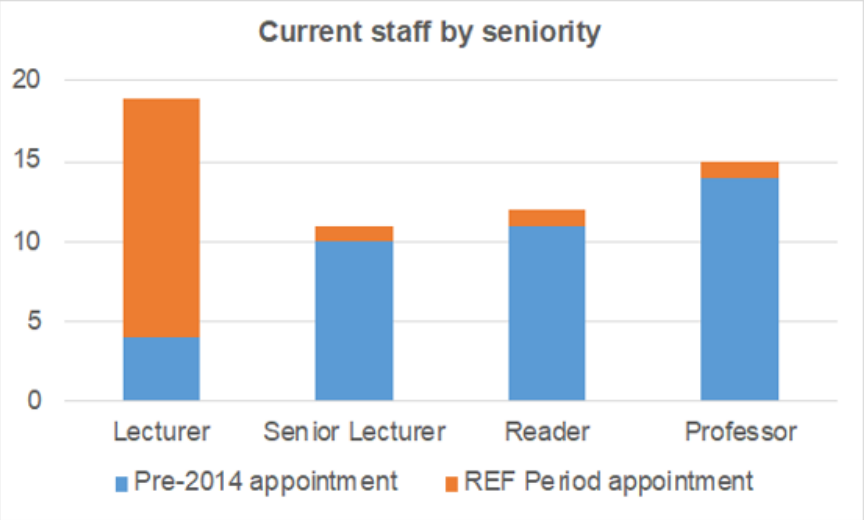
People are the indispensable heart of the department. We encourage and enable the development of all staff, from research students to professors. Of the 57 category 1A staff, 53 are on open-ended contracts, 3 are fixed-term with opportunity for renewal/transfer to open-ended, and 1 fixed-term. We have strengthened the department through investment in new academic staff (approximately 30% increase since 2014). We have appointed excellent ECRs and prestigious fellows, e.g. UKRI fellow Schaich (Fundamental Particle Physics) and EPSRC fellow Rizzardo (Pure Mathematics), developed the *Institute of Financial & Actuarial Maths* (IFAM – established 2011) to be the best in the UK for Actuarial Research, and initiated two exciting new research areas, *Stochastics* and *Mathematics for Health*.



*Reconfigured as Applied and Stochastics in 18-19.
**Reconfigured as Fundamental Particle Physics and String and Beyond-Standard-Model Phenomenology in 16-17.

2.1.1 Recruitment

Principles: We hire outstanding individuals to support and develop sustainable research clusters (S1) and empower staff to conduct outstanding research (S2): ECRs in well-established clusters and research leaders to guide the formation of newer ones.



We actively recruit staff who enhance collaborative and interdisciplinary activity (S3) with other disciplines, e.g. health and finance, but also within mathematics e.g. at the interface between algebraic geometry and string theory. These, and additional appointments aligned with major grants (LCMH, UKRI FLF), strengthen our connections with the priority research themes of UK Research Councils (S4). Applicants discuss non-academic beneficiaries of their research during the selection process as part of our intensified focus on impact and the promotion of mathematics to the wider public (S6). The department embraces Faculty and University ED&I principles including two-tick inclusivity. We aim to have this recognised in a 2021 Athena Swan Gold submission by the School of Physical Sciences (S5).

Financial and Actuarial Mathematics: The *Institute of Financial & Actuarial Mathematics* is at the forefront of the contemporary interest in applications of mathematics to insurance and finance. We balance appointments between fundamental research in applied probability and stochastic processes and outward-facing applications. We have appointed ECRs in Financial Mathematics (Deshpande [2014], Rojas Nadayapa [2016], P Eisenberg [2017]) and Actuarial Mathematics (J Eisenberg [2017], Zhu [2018]). Recently we strengthened the interface between probability and applications by appointing Patie [2020] in a senior position supported by Azmoodeh [2020].

Stochastics: Establishing a new *Stochastics* research cluster, incorporating the existing *Statistics & Probability* group, was a priority. Konstantopoulos [2018], formerly Chair of Mathematical Statistics at the University of Uppsala, was appointed to develop the cluster, supported by Reader Li [2018] from Shanghai Jiao Tong University, and ECRs Yuan [2020] and Ojeda [August 2020]. The cluster conducts research in Stochastic Processes, Probability Theory, Statistics, Operational Research and Random Structures. It will coordinate our contribution to the mathematics of Data Science, foster collaborations between departmental groups using stochastic methods (e.g. in IFAM and FPP), and engage with cognate research groups in other departments, e.g. Maskell's *Algorithmic Group* in *Electrical Engineering and Electronics*. The cluster has already established collaborations with universities in Europe, Russia, the US and Australia.

Pure Mathematics: EPSRC Fellow Rizzardo [2017] strengthens the *Algebraic Geometry* (AG) group through her pioneering work on derived equivalences. The group hosted two major conferences in 2019, *British Algebraic Geometry* in the spring and *Geometry of Derived Categories* in the autumn. Mertens [2020] broadens the scope of AG with his work on modular forms and their applications in *Number Theory*. Meyer [2017] strengthens the department's expertise in analysis; working at the intersection of dynamical systems, analysis on (fractal) metric spaces, and geometric group theory, connecting researchers in Dynamical Systems with those in Geometry & Topology and Stochastics.

Dr. Rizzardo (EPSRC Fellow, 2016-19)

Applied Mathematics: Appointments enhance the existing areas of excellence with a new emphasis towards data science, adding value to EPSRC major grants on Metamaterials & Healthcare Technologies. Domijan [2016] expands the work of the Mathematical Biology group towards systems biology and dynamic networks. Colquitt [2016] strengthens the Waves and Mechanics group with expertise on the dynamic response of elastic structured media (metamaterials'), and Haslinger [2020] is an expert in flexural waves in multi-scale structured plates, resonance transmission, localisation and trapping, with applications to non-destructive evaluation of defects in solids. Valkonen [2016] and Alpers [2019] bring a new focus on the application of functional analysis and PDEs to image processing and data science.

Theoretical Physics: Hardy [2017] brings expertise in dark matter and is leading the Theoretical Physics Component of the *Quantum Sensors for the Hidden Sector* project (a collaboration of 7 UK institutions applying quantum technologies to search for dark matter); Parameswaran [Marie-Curie Fellow 2015-17, then Lecturer] relates string theory to cosmological data and is renowned for her work on inflationary scenarios in string theory. Langfeld [2016] was recruited as HoD and to bring a new computational emphasis to Lattice Gauge Theory, supported by ECR Schaich [2019] (UKRI Future Leaders Fellow) and Buividovich [2019]. Schaich's computer simulation expertise centres on nearly conformal field theories and supersymmetric extensions of the Standard Model. Buividovich broadens our impact potential through his simulation of QFTs underpinning solid state physics, especially those focused on graphene.

Dr. Schaich (UKRI Fellow, 2019-26)

Centre for Enhancement in Education: A key strategic evolution since REF2014 is the appointment of six 'Teaching and Scholarship' academics who develop innovative resources in order to free academic time for research (particularly beneficial in the COVID-19 crisis). These appointments contribute to impact through developing research-connected teaching and industrial links building upon the IFAM summer research internships (§4.2.4).

2.1.2 Staff Development

The department's strategy for career development (S6) implements the university's commitment to the principles of the Concordat to Support the Career Development of Researchers (2020 renewal of HR Excellence in Research award). The development of PGR students is discussed in §2.2 and that of a new generation of mathematical researchers in §4.3 and §4.4.

Mentoring and support: We support research staff via a two-fold mentoring scheme. A departmental mentor offers new staff support on subject-specific issues such as publications and grant writing. The University mentoring scheme offers an online database of extra-departmental mentors who provide a broader view on career strategies.

We support probationary lecturers by reducing teaching load by 50% to create space to build an independent research profile, apply for grants, and complete the training provided by the Department, School and Faculty. They receive grant writing support from a senior colleague within their cluster and advice from the Impact Group to ensure impact is embedded from the outset. The Department provides letters of support for grant proposals and, when these are successful, a further reduction in teaching and administrative load.

We fund regular cluster seminars, enabling staff to communicate and collaborate with academics from other institutions. The clusters provide an informal support network, overlaid by Department, School and Faculty networks. The departmental "ECR coffee morning" provides peer-to-peer support with input from the HoD, senior staff and Impact Officers, and a forum for networking, raising questions and providing feedback to the HoD.

The School and Faculty Research Support Offices advise on grant proposals and inform staff of funding opportunities, and potential connections with UKRI priority themes. This support is supplemented by the new departmental Research Support group (§1.3.2). We encourage academic staff to organise and contribute to research meetings across faculty boundaries to facilitate impact e.g. the EPSRC network funded 2015 workshop co-organised by Bearon and Webb (Pharmacology) contributed to the LCMH application. The Research Centre in Mathematical Modelling enhances collaborative and interdisciplinary activity by funding and hosting group meetings with external colleagues.

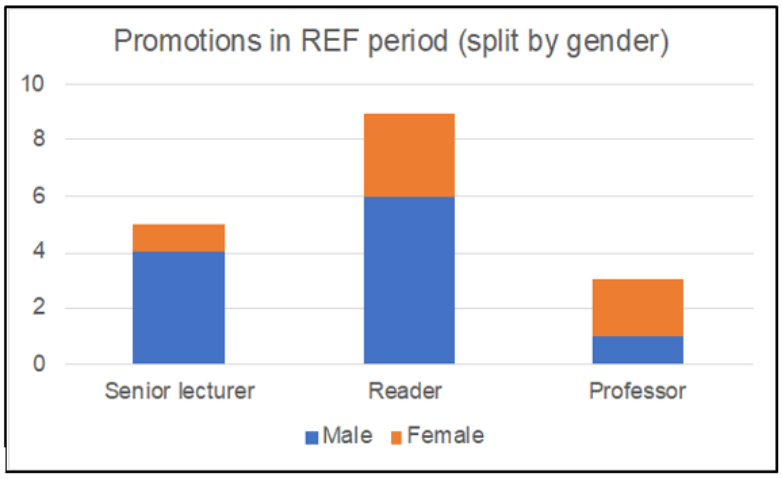
Review and Development: Staff, including PDRAs, undertake an annual Personal Development Review (PDR) with a senior colleague. This covers contributions to teaching, research, impact, and administration and is an opportunity to discuss personal development and career plans, including promotion, and identify the support required to achieve these. Engagement with the PDR is excellent with almost 100% completion each year.

We encourage staff to develop their research profile by attending conferences and meetings and publishing in highly-regarded journals. We provide new staff with travel and conference support, and prioritise them when allocating PhD studentships (six REF-cycle appointments are primary supervisors at census date). The department supports staff with research leave and by allowing them to concentrate teaching in one semester to free up focused research time.

The University's Professional and Academic Development Academy provides bespoke personal development training. For example, staff, including the HoD and ECRs, are participating in the PROSPER project, which provides training for research staff and PIs, and in the newly launched Heilbron programme for senior leaders. Successes include Boado Penas – promoted to senior lecturer and appointed lead liaison with the Institute and Faculty of Actuaries after completing Advance HE Aurora leadership development – and Sharkey – appointed as Research Committee chair after completing Research Team Leadership training.

Confirmation and Promotion: Confirmation in appointment, promotion and annual salary review are clearly structured and instigated by HR. The HoD meets all probationary staff prior to their

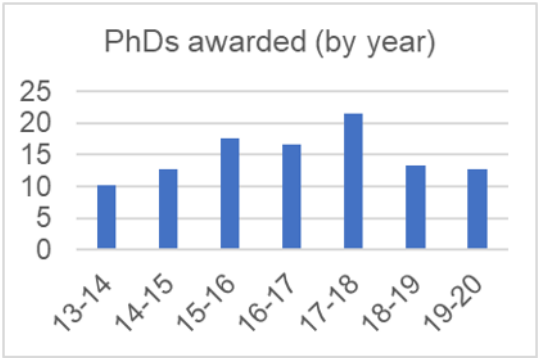
confirmation interview, and all staff considering promotion, to discuss their aspirations and advise on process. Within the REF period 17 staff have been promoted to senior lecturer or higher.



We support continuity of employment for PDRAs – e.g. Liverpool Centre of Mathematics in Healthcare PDRAs Hughes and Leedale now hold MRC Skills Development Fellowships at UoL – and transition from fixed-term to permanent employment – e.g. Marie Curie Fellow Parameswaran was appointed lecturer in 2017.

2.2 Research Students

2.2.1 Overview: Since 2014 we have recruited an average of 18 PGRs per year and contributed to the supervisory team of others within the University but outside the department, as well as 7 at the African Institute of Mathematical Sciences (§4.3.3). Altogether 104.88 FTE PhDs have been awarded (counted by fraction of supervision we provide).



Joe Siddons (PhD 2018) PDRA, Marine Macroecology and Biogeochemistry, Dalhousie University

Jaclyn Bell (PhD 2018) Senior Teaching Fellow, Imperial College, and UoL Alumnus of the Year 2019

Mike Roberts (PhD 2018) PDRA, Image Analysis, DAMTP, Cambridge, and Post-Doctoral Fellow, Astra Zeneca

2.2.2 Support: Cluster PGR leads support the departmental PGR Director by providing subject-specific guidance. Together they monitor student progress and satisfaction, liaise with students regularly e.g. in the Student Staff Liaison Committee (SSLC), facilitate PGR recruitment and support students in organising events such as the weekly PGR Coffee morning.

The PGR director develops action plans with the SSLC in response to feedback from the PRES. Since the 2017 PRES and Action Plan, our scores have significantly (>10%) improved in key areas such as `I am aware of opportunities to become involved in the wider research community'; `I have access to a good seminar programme in my research area'; `I understand my responsibilities, and am aware of my supervisor's responsibilities towards me'.

2.2.3 Funding: The department receives approximately 4 DTP studentships from STFC and EPSRC per year. The Department co-leads (Co-I: Pantelous) the EPSRC CDT in Risk and Uncertainty (8 students). We have attracted industrial CASE studentships, e.g. on *Automatic CT Segmentation of the Lungs* with Liverpool Heart and Chest NHS Foundation (Chen) and further PhD funding through EU ITN grants (CERMAT2 and Renal Toolbox).

Since 2014, the School of Physical Sciences has funded approximately 5 PGRs per year. These come without nationality restrictions, allowing us to recruit excellent international candidates. Our main criterion is academic quality, and our main principle is to create a science-led community to strengthen our international research profile. Since 2016 we have jointly supervised approximately 4 students per year with our Chinese partner University XJTLU. We also have 4 students registered on a joint PhD programme with National Tsing Hua University, Taiwan and have recently initiated another joint programme with Capital Normal University, China (§4.3.4).

2.2.4 Recruitment: Projects are advertised via the University's PhD project website and externally. We attract students from diverse backgrounds from all over the world (the 59 current PGRs come from 18 countries across 4 continents). As part of our wider career strategy, we nurture a pipeline of PhD students from our undergraduates by ensuring that final year MMath, MPhys, and MSc students conduct a research project within one of the clusters.

We follow central guidance on ED&I from recruitment to graduation; PGR students help shape policies and are represented on the School ED&I group. We support students transitioning to part-time/flexible working and provide leave for maternity and short-term external research placements.

2.2.5 Training: PGR students are full members of their cluster and participate in its research activities. Less formal activities such as the "friendly learning seminar on Fourier-Mukai transforms" run by Rizzardo attract PGRs from the wider Department, not only those from *Algebraic Geometry*. In addition to cluster-specific activities, PGRs are fully included within the Barkla and Terry Wall Lecture series which provide them with the inspirational opportunity to listen to, and speak with, some of the foremost mathematicians and theoretical physicists of the day (§4.4.4). Our research support budget provides funding for conference and workshop attendance and external training, enabling PGRs to integrate with their global research community.

Broader training is provided via MAGIC – a consortium of 21 Universities providing video lecture courses. Currently 35 MAGIC courses are available, spanning a wide range of topics; we have contributed four (in dynamics, geometry, singularity theory and string theory) in this REF period. Students in the Stochastics cluster also attend APTS residential courses in probability and

statistics. IFAM collaborates with the Risk Institute to run an annual spring school for PhD students in the CDT in Risk & Uncertainty attended by >50 students from Liverpool and UK HEIs.

Training and experience in teaching develops key transferable skills – communication, leadership, team-work and interpersonal skills – as well as being important for PGRs progressing to academic careers. PGRs can choose to give tutorials (jointly with academic staff), assist in marking, and/or help at Open and Applicant Discovery days, workshops and conferences. They can also assist with the Summer Internship Scheme (§4.2.4), which involves liaising with industrial partners as well as supervising UG students. We provide training and all work is paid, with hours limited to preserve time for study. In 2016 the University created a pathway for PGR teaching assistants to obtain accreditation as Associated Fellows of the HEA (3 PGRs accredited, 5 being mentored).

2.2.6 Support, review and completion: Each PhD student has at least two academic supervisors, at least one of whom is senior. This ensures quality of supervision and provides informal training, in addition to the formal university training, for less-experienced supervisors.

Each student undertakes a Development Needs Analysis within their first three months. The results are included in a Training and Assessment Plan with specific objectives including training, relevant conferences and dissemination targets for results. Plans are updated at annual review, which involves presenting at a seminar, submitting a detailed report and a panel interview. Second year PGRs present a poster at a Faculty-wide session.

Well-Being Ambassadors provide additional support and local administrative staff hold drop-in sessions which sign-post support services such as counselling and mental health advice.

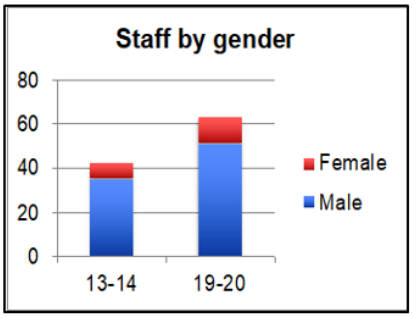
2.2.7 Independence: The Department actively encourages, and financially supports, the independence and self-governance of our PGR community. Amongst other events PGRs organise the School PGR Forum, and a weekly Departmental PGR Coffee Morning.

In 2019, three of our PGRs, jointly with one from Electrical and Electronic Engineering, organised the 3-day conference “Early Career Researcher Conference 2019”, with almost 80 attendees, focusing on Data Science, Machine Learning and Artificial Intelligence.

2.3 Equality & diversity

Equality, Diversity and Inclusion (ED&I) underpin our Strategic Priorities S1-S6 and are interwoven with all our activity. Our open, supportive and inclusive environment was recognised by an Athena SWAN Silver award for the School of Physical Sciences in 2016, following a Bronze award in 2014. Staff, including the HoD and ECRs, are participating in the University’s *Research in an Inclusive and Sustainable Environment* (RISE) project, launched in response to the profound and differential impact of COVID-19 on research.

2.3.1 Gender equality: Historically women have been under-represented within mathematics, particularly at PGR and above. An important priority for us has been to improve the gender balance through appointments, flexible working policies to aid retention, and improved opportunities for career progression. One quarter of current (July 2020) staff appointed during the REF period are female. Of six non-professorial female staff in post in 2014 Bearon and Constantinescu are now professors, Pratoussevitch is a reader, and Boado Penas a senior lecturer.



We promote gender equality from the ground up. The Outreach Team organise an annual Girls in Maths event encouraging girls to take A-level maths with contributions from female staff (e.g. Bearon, Pratoussevitch, Zychaluk), students and alumni. We contribute to the Liverpool Women in Science & Engineering Christmas lectures (e.g. Constantinescu, 2016). The new HoD (Bearon) and head of IFAM (Constantinescu) are excellent female role models for students and colleagues.


Prof. Bearon (HoD) and Prof. Constantinescu (Head of IFAM)

2.3.2 Protected characteristics: We recognise that ED&I encompasses more than gender equality. The Department engages positively with the University BAME, Disabled and LGBT staff networks. These are promoted to staff during induction and through ED&I events such as the 2019 LGBT Event "No Sexuality Please, We're Scientists". We are involved in and co-organise School-wide events such as Celebrating Success in Physical Sciences (featuring Menoukeu-Pamen in 2019 and Parameswaran in 2020).


Dr. Menoukeu-Pamen and Dr.Parameswaran

The Outreach Team promotes mathematics in the wider community (S6) and aims to reach as many children from under-represented backgrounds as possible. The new University of Liverpool Mathematics School (§4.4.2) augments this.

In order to create a supportive and inclusive environment (S5), we organise a number of community and Well-Being events including talks by external speakers, Tai-Chi and health MOTs. Since 2019 we have a well-being room where staff can take a break and relax.

2.3.3 Recruitment and progression: We embed ED&I considerations in the recruitment process through the Faculty Good Practice Guide which covers all aspects from advertisement, interview panels, to appointment. Potential adjustments include part-time working at the start of the employment (e.g. J Eisenberg).

We encourage all staff to consider preparing for promotion in their annual PDR. This can include specific training and development for under-represented groups, such as the Aurora Programme for women in academia (e.g. Constantinescu, Boado Penas). The University Mentoring Scheme provides a range of support including leadership training for BAME academics. Senior female staff (Bearon, Constantinescu) identify this scheme, and Faculty & University Leadership programmes, as supporting their promotion to Reader and then Professor during this REF period.

2.3.4 Flexible working: We promote work-life balance; staff are supported to work flexibly when necessary, or desired, to look after children (e.g. Bearon, Zychaluk, Rempe-Gillen, J Eisenberg), or to prepare for retirement (Rees). These arrangements can be made either permanently or temporarily, e.g. Bearon returned to full-time work after 6 years part-time. Flexible working allows staff to adjust their working pattern to their needs. Staff with caring responsibilities can request that their teaching is in restricted hours. We schedule departmental meetings in core working hours (10am-4pm) to enable staff with caring responsibilities to attend.

Prof. Rempe-Gillen (0.75 FTE for childcare)

Staff returning from long periods of leave discuss their individual needs with the HoD and are offered a package which may include a lower teaching or administrative load. Our Family Working Adviser provides information on adjustments to working pattern, leave, and access to services for parents e.g. nursery and carers' car park. PDRAs and PGR students are eligible for flexible working and financial support for maternity/parental leave even if not provided by the funding body; e.g. Semertzidou had 6-month maternity pay in 2016 funded by the School.

2.3.5 Equality and diversity in REF submission preparation: The departmental REF panel had oversight of all REF selection activities and ensured compliance with the University REF Code of Practice. All members undertook Advance HE half-day training on equality, diversity, inclusion, and unconscious bias. Particular attention was paid to the panel's diversity; 4 of 12 members were female, two ECRs and one an independent external observer.

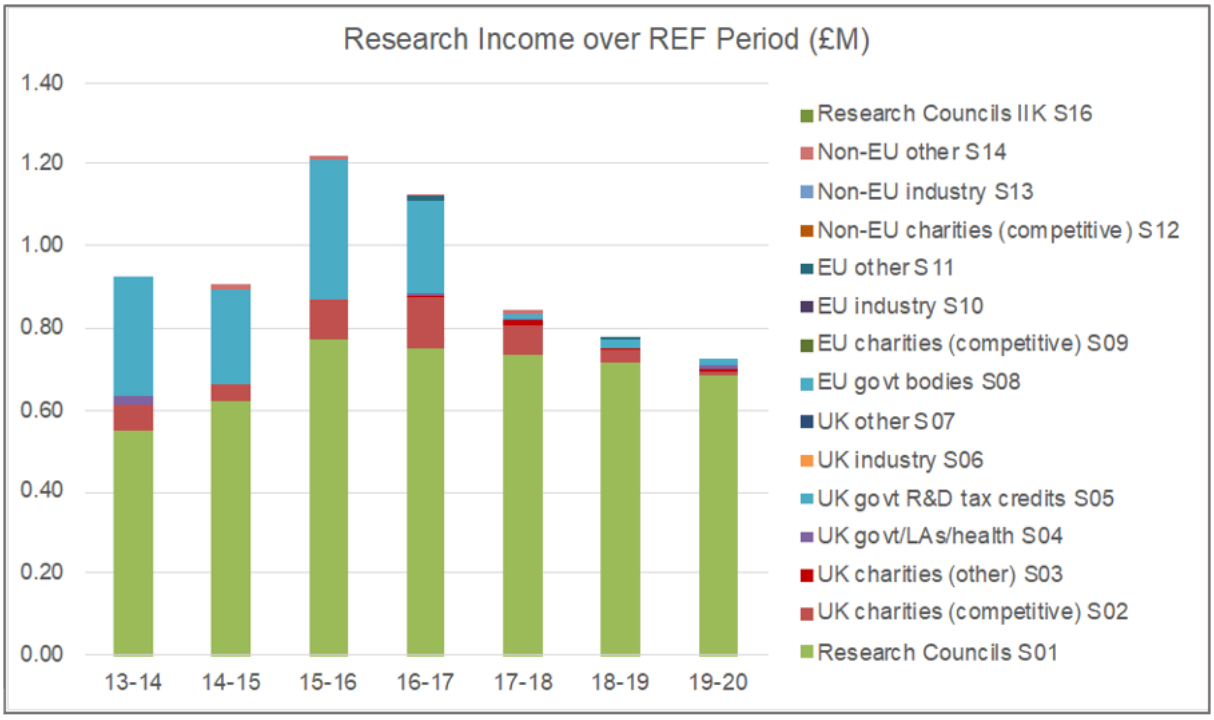
3. Income, infrastructure and facilities

3.1 Research Funding and Strategy

3.1.1 Overview: This is driven by our strategic priorities: empowering staff to conduct outstanding research (S2); enhancing collaborative and interdisciplinary activity (S3); developing a new generation of researchers (S6). We support staff to generate research income through

RF1 Leading and contributing to large interdisciplinary collaborations,
RF2 PI-driven fundamental research in core strengths,
RF3 Securing funds for workshops, networks & travel.

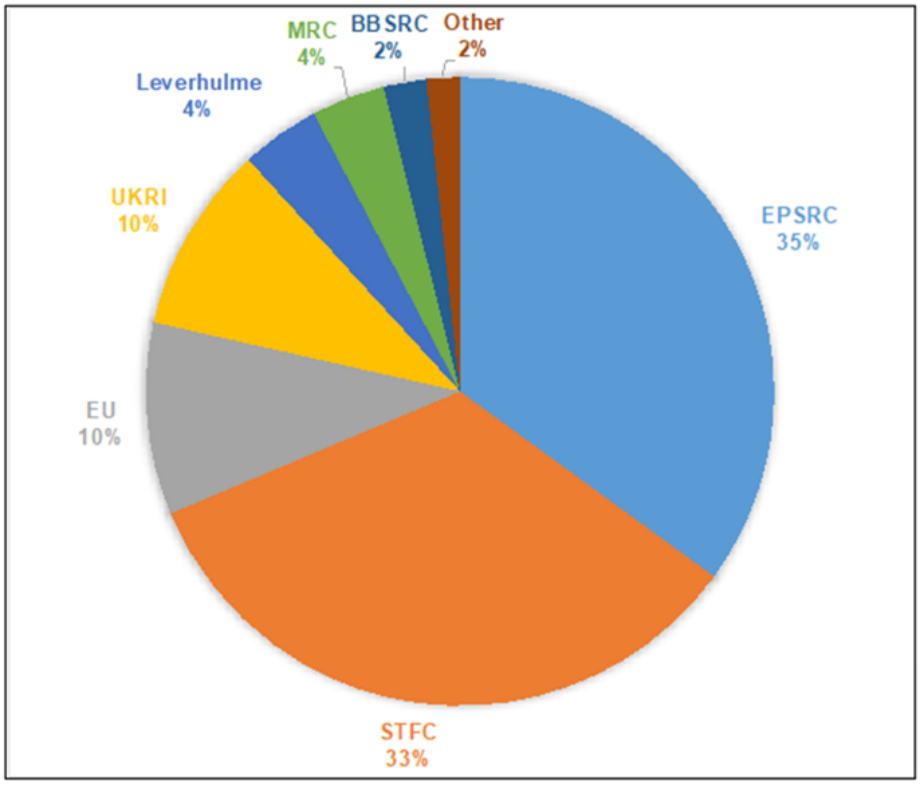
We continue to focus on fundamental research supported by our main funding sources (EPSRC and STFC) through strategic recruitment (§2.1.1) and support and development of existing staff (§2.1.2). We have leveraged several large (>£1M) grants during the REF period by aligning proposals to UK Research Council priority themes (S4) and utilising the expertise of institutional professional support. We have successfully secured funding from diverse agencies, enabling the development of global collaborative networks.



Total research income was £6.5M in this REF period. UK Research Council income increased, but EU income has declined since 2016. To mitigate, we have restructured Research Committee (§1.2.3) to enable us to diversify our funding sources and increase future income.

For grants awarded in this REF period, the total investigator funder contribution was £7.8M (increased from £6.0M in the previous period, see below for breakdown by source), the total value of PI-led projects was £9.3M, and the total funding volume of research projects on which we collaborated was £33.4M.

We will use the enhanced support provided by the new Research Support group (§1.3.2) to continue to bid successfully for UK Research Council funding and to increase the number of small grants obtained for meetings, workshops and travel, in particular targeting grants from LMS Schemes 2 (Visits to the UK), 4 (Research in Pairs) and 5 (Collaborations with Developing Countries). This will supplement our current successes in obtaining funding from LMS Scheme 3 (Networks), the Royal Society, Leverhulme Trust, IMA, India Partnership Development Fund, ODA Research Seed and GCRF Rapid Response Funds.



Grants awarded (investigator funder contribution)
In REF period by source (total = £7.8M)

3.1.2 Large interdisciplinary collaborations (RF1; S3, S4)

Liverpool Centre for Mathematics in Healthcare: In 2016 we established the LCMH (S2) through an EPSRC large grant (PI: Chen, 2015-2020, £2.5M). The proposal built on the expertise of the *Centre for Mathematical Imaging Techniques* (CMIT) in variational models and applications of imaging to restoration, tomography, segmentation, registration and deep learning, and of the *Mathematical Biology* research group, which undertakes research on stochastic and deterministic spatial-temporal dynamics models in a range of biological systems including disease spread. Other key factors were the strength of the Faculty of Health & Life Sciences, the University's links to the Liverpool Health Partnership, and support from the University Research Office.

The successful bid built upon prior major collaborations in modelling biomechanics of tissues supported through a large EPSRC grant (EP/K036939/1, PI: Chen, 2013-17, £1.3M), and funding awarded to members of the *Mathematical Biology* group by BBSRC (BB/M026434/1, PI: Sharkey, 2015-17, £216k: BB/K002430/1, PI: Vasiev, 2012-16, £254k) and the Leverhulme Trust (PI: Sharkey, 2015-18, £163k).

LCMH has contributed to successful funding bids led by the *Mathematical Biology* group: MRC Skills Fellowship for LCMH PDRA (MR/S019332/1 PI: Leedale, 2019-21 £236k); EPSRC standard grant on early stage biofilm formation (EP/S033211/1, PI Bearon, 2020-23, £361k); EU ITN `Renal Toolbox' (Co-I: Bearon, 2019-21, Total £0.8M). In addition, £6.6M was awarded to associated investigators by BBSRC, CRUK, EPSRC, MRC, Cancer Research UK and Kidney Research UK.

Theoretical Physics: With significant overlap of the research interests and direct collaboration between Theoretical and Experimental Physics in the areas of flavour and precision lepton physics and dark matter, we have been an integral part of recent STFC Particle Physics Consolidated Grants held in the Department of Physics: Theoretical Particle Physics, (PI: Langfeld, Total funding: £0.9M); Experimental Particle Physics, (PIs: Bowcock & Vossebeld [Physics], 2019-22, £8.3M) includes Gorbahn and Teubner as Co-Is (awarded £265k). The Theoretical Particle Physics bid involved 15 staff across the two research clusters in Theoretical Physics, including ECRs, Hardy, Parameswaran and Schaich.

Metamaterials: Collaborative grants awarded to the Waves and Solid Mechanics Group within, or overlapping with, the REF period include an EPSRC Programme Grant – Mathematical Foundations of Metamaterials for Multi-Scale Physics and Mechanics (Co-Is: A&N Movchan, 2014-20, £489k of total £2.6M) – and three EU grants – New Ceramic Technologies (PI: N Movchan, 2013-17, £238k of total £398k), Modelling and Optimal Design of Ceramic Structures (PI: N Movchan, 2013-15, £97k), Component Fragility Evaluation and Seismic Safety Assessment of Chemical Plants (PI: A Movchan, 2014-17, £60k of £130k total).

3.1.3 PI-driven fundamental research in core strengths (RF2; S1, S2): We encourage novel fundamental research through staff appointments and ensuring the vitality of our clusters. Evidence of success includes fellowships (J. Eisenberg, Gracey, Leedale, Menoukeu-Pamen, Parameswaran, Rizzardo and Schaich) and substantial buy-out of staff time for research. Over the REF period, ECRs (Hardy, Parameswaran, Rizzardo, Schaich, Valkonen) have been awarded £1.9M of research council funding.

3.1.4 Workshops, networks & travel (RF3; S2, S3, S6): We encourage staff to seek grant income for building networks. These directly contribute to outstanding research outputs, e.g. Hall's 2018 EPSRC Overseas Travel grant facilitated his collaboration with Profs Boyland (Florida) and de Carvalho (São Paulo), resulting in papers in *Inventiones Mathematicae*, *Trans. AMS* and *Geometry & Topology*.

Constantinescu (IFAM) coordinated the Risk Analysis, Ruin and Extremes (RARE) project (EU FP7 no. 318984, 2012-16, 782k). RARE connected 12 HEIs and >60 researchers globally working on extreme events and their applications to insurance modelling. It resulted in >200 publications, including a special issue of the Annals of Actuarial Science, and >10 conferences and workshops. RARE enabled extensive secondments of academics and PGRs to partner institutions including ETH Zurich, ESSEC Paris, ISI Calcutta, Monash University, and Nankai University. These collaborations continue to benefit the department: HEC Laussanne co-organises the biannual PARTY conferences, ESSEC Paris is a partner in a new large grant application, and we have initiated a dual PhD programme with Nankai University (§4.3.4).

3.2 Organisational Infrastructure

3.2.1 Clusters: The six research clusters provide research environments of a suitable scale to enable collaboration. Each consists of staff with related research interests co-located within an area of the department to maximise opportunities for informal discussion. To maintain clear research foci Theoretical Physics was organised into Fundamental Particle Physics and String- and Beyond-Standard-Model Phenomenology, each receiving two junior appointments (Buividovich, Schaich and Hardy, Parameswaran respectively) to ensure sustainability. Likewise,

Applied Mathematics, Statistics and Probability was reorganised into Applied Mathematics and Stochastics, with respectively four ECR appointments (Alpers, Colquitt, Haslinger, Valkonen) and two senior appointments (Konstantopoulos, Li) supported by ECR lecturer (Yuan).

3.2.2 Research Centres: The Research Centre for Mathematical Modelling (RCMM) initiates and co-ordinates research visits & workshops across the department. Our interdisciplinary research centres (LCMH, CMIT & ERRC) develop and promote impact and applications. Staff also contribute to UoL interdisciplinary centres (e.g. Pantelous co-I on Risk Institute CDT). The Research Committee oversees and promotes external opportunities.

3.2.3 Impact Group: Two Impact Coordinators – senior academics Sharkey and Chen – and 1FTE departmental academic Impact Officer – shared between two 50% part-time officers each appointed for 3 years from 2019 – provide bespoke support in the form of project timelines, action plans and facilitation and identification of new impact.

Flexible funding is provided from a number of sources including the Higher Education Impact Fund (HEIF), the EPSRC Impact acceleration account (IAA), as well as school and department funding.

The Impact Officers track impact activities and non-academic partners, providing staff with a database of directories and expertise. They share best practice and disseminate opportunities such as hackathons, impact-related funding and training via a weekly email. The Impact Coordinators led a focused discussion at the 2017 Staff Away-Day and, with the HoD, emphasise the importance of impact at recruitment and annual PDRs.

3.2.4 Grant support: Currently, we organise grant support at cluster level, ensuring it is subject-specific with strong peer-to-peer support. We also actively contribute to, and benefit from, institutional support, such as internal peer review and preparatory interviews. This strategy has enabled us to increase the number, value and success rate of applications: 11/24 applications succeeded in 2013-14 (£0.97M awarded of £5.45M investigator funder contribution applied for), compared with 37/63 in 2018-19[2] (£5.16M of £12.41M investigator funder contribution).

We created a new academic lead and working group for Research Support as part of the 2019 restructuring of the Research Committee. Their role is to supplement the cluster-level support, propagate good practice between clusters and facilitate inter-cluster proposals.

3.3 Operational Infrastructure:

3.3.1 Maths building: The Department is located in the centre of the UoL campus. Co-location with research units and centres within the University and The Liverpool Royal Hospital facilitates interdisciplinary collaboration. For example, LCMH includes co-investigators from six UoL departments; there are strong interactions with Physics (§3.1.1); involvement in the Risk Institute has allowed us to train a strong interdisciplinary cohort of PhD students.

To create a physical environment to support our strategic aims we have invested in our building over the REF period. We refurbished the seminar rooms to provide state-of-the-art facilities for delivering and recording mathematical talks, with timetabling priority for research seminars. We created space and offices for the new Stochastics cluster, an open plan office for LCMH research

[2] Last year with complete data; some 2019-20 application outcomes undecided.

staff, and new offices to host all IFAM staff on a single floor and to accommodate the increasing staff numbers in the theoretical physics clusters (both housed in the same wing).

All PhD students, PDRAs, and Impact Officers are located within the mathematics building to maximise opportunities for discussion and collaboration. We have two common rooms with kitchen facilities to encourage staff and PGR interaction, and foster an inclusive, collaborative environment. We also have the Fröhlich meeting room & library, and two visitor rooms in the Centre for Mathematical Imaging Technology and in the Research Centre for Mathematical Modelling with open plan desk space for research visits.

3.3.2 Environmental Radioactivity Research Centre: The ERRC's principal facility is the Environmental Radiometric Laboratory, situated in the Oliver Lodge Building, developed jointly with the Department of Physics. The laboratory leads the world in use and development of low-background hyper-pure germanium gamma spectrometers for measuring low-level environmental radioactivity.

3.3.3 High-performance computing: The Department has access to the University's central High-Performance Computing facility BARKLA and High Throughput Computing facility under the Condor setup. BARKLA is a tightly coupled computer cluster with all nodes connected by 40 Gbit/s Infiniband. We used a mix of School and Schaich's UKRI Fellowship funds to purchase 7 new nodes as part of its 2019 expansion which gives us priority access to 15 of the 138 nodes, each with 40 cores and 384GB of memory.

Rakow and Schaich's research benefits from cross-HEI shared, collaborative research infrastructure. Rakow was Co-I on STFC DiRAC projects using the DiRAC BluGene/Q and Tesseract in Edinburgh as well as Cambridge's DiRAC Data Intensive Service. A total of 343 Mhours core hours were allocated to these in the REF period. In the same period Schaich, as a Co-I, had access to NSF and DoE computing facilities in the USA equating to 244M core-hours.

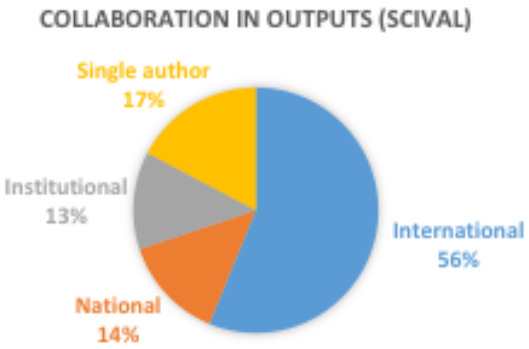
4. Collaboration and contribution to the research base, economy and society

We are guided by our strategic priorities: to empower staff to conduct outstanding research (S2), to enhance collaborative and interdisciplinary activity (S3) and to promote mathematics to the wider public and develop a new generation of mathematical researchers (S6). Support includes:

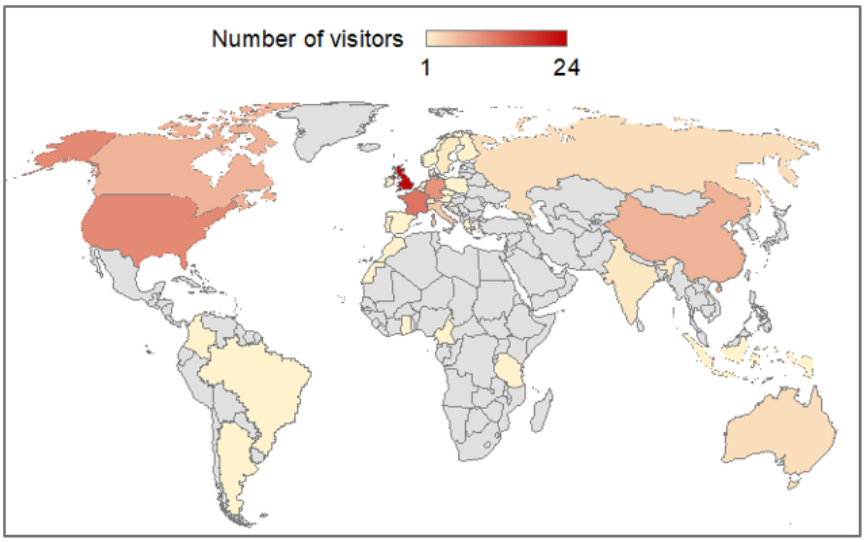
- Resources: the Research Centre in Mathematics and Modelling allocates pump-priming funding; the Department provides physical space for hosting networks, workshops and conferences.
- Training and Advice: the Impact Group advises on development of impact. The central training team, `The Academy', and the School of Physical Sciences offer skills training workshops to ECR and PDRA staff on teamwork, project management, scientific writing, collaboration, grant writing, and peer review (§2.1.2).
- Time: Research and impact leave (§2.1.2); investment in T&S staff frees staff time for research (§2.1.1).
- Legal: Research Partnerships and the Innovation and Legal departments facilitate formal links where appropriate: Memoranda of Understanding are used for joint research activities (§4.3.3) and PhD schemes (§4.3.4).

4.1 Research collaborations, networks and partnerships

Research in Mathematical Sciences at Liverpool is highly international and collaborative. Of the 538 outputs authored in 2014-20 by UoA Academics, 313 involve international co-authors and a further 69 have external co-authors from the UK; together these constitute 71% of our outputs (SCIVAL).



4.1.1 Collaboration: We have increased support for collaboration through an enhanced visitor programme, doubling the number of visitors from the previous REF period to over 150 (excluding workshop / conference attendees), coming from 122 HEIs or research institutions within 37 countries. We achieved this increase by encouraging and supporting applications for small grants to fund visitors from the LMS, EPSRC, INFN (Italy), ICMS, The Royal Society, Newton Fund, the British Council, eight EU Marie-Curie Actions grants, and industrial partners.



Global distribution of visitors during REF period (departmental data).

The Research Centre for Mathematics and Modelling provides match-funding where necessary to secure grants, and funds small-scale high-value activities outside the remit of external funders or occurring outside the timeframe of funding rounds.

4.1.2 Networks: Several research groups organise and participate in larger networks, e.g. the Risk Analysis, Ruin and Extremes project (§3.1.4).

Phenomenologists from the Fundamental Particle Physics group collaborate with experimental groups (REF panel B9), including CERN and Fermilab, USA. Vogt is part of a team that pushes precision strong-interaction physics to the accuracy necessary for current and future colliders. Teubner and Gorbahn provide theoretical expertise on Standard Model calculations. Gorbahn and Gracey are the UK representatives on the management committee of the European Cooperation in Science and Technology action `Unravelling new physics at the LHC through the precision

frontier'. Teubner is also a participant and, since 2013, full member of the g-2 experiment at Femilab (US). He is directly supported by the STFC Particle Physics Consolidated Grant. The group participates in the STFC-funded Virtual Centre for UK Lattice Field Theory (UKLFT). Schaich leads (jointly with Bane from Computer Science) the UoL Quantum Computing Network.

In 2018 Gracey was awarded a DFG Mercator Fellowship, recently renewed for a further two years, and held a CNRS Visiting Fellowship at the Sorbonne, Paris. He also collaborates with condensed matter theorists on quantum critical points in models of graphene.

The pure cluster is an active participant in LMS Scheme 3 funded research networks. Goryunov organised the *British Singularity Days* (2014-2018), Rizzardo is the local organiser for the *GLEN (Glasgow-Liverpool-Edinburgh-Newcastle) Algebraic Geometry Network* and Woolf for the *Transpennine Topology Triangle*. The department sponsors meetings by providing facilities and contributing to expenses. Members of the pure cluster also participate in the *Applied Algebraic Topology* network jointly organised by Kurlin (Computer Science, UoL).

4.2 Key research users and beneficiaries

We channel inter-disciplinary, impact-related and industrial collaborations through our research centres and the Institute for Financial and Actuarial Mathematics. These are aligned with the focus of our impact strategy on health, environmental modelling, metamaterials, and financial and actuarial applications; illustrated in impact cases ICS1-3,5. This strategy has resulted in a marked increase in the number of outputs co-authored with corporate partners from just 1% in 2014 to 8% in each of 2019 and 2020 (Scival).

4.2.1 Liverpool Centre for Mathematics in Healthcare: LCMH conducts multidisciplinary research to explore how mathematics and statistics can deliver accurate predictive models and tools for personalised healthcare delivery. It brings together mathematicians, scientists, engineers and clinicians from the Universities of Liverpool and Lancaster with industrial partners and policy makers who interact with 15 other UK universities and 40 NHS trusts. Eight academics from the pure and applied clusters are Co-Is, alongside 13 further Co-Is from UoL.

LCMH has established collaborations with Liverpool Health Partners (LHP), six local hospitals, Public Health England, AstraZeneca (Global), Mirada Medical UK, Carl Zeiss SMT Ltd, Unilever, Bionow Ltd and Ultravision. Key impacts:

- Collaborative work with Alder Hey Children's hospital and TRITEC Developments Ltd to track and analyse movement around a children's ward in conjunction with tracking bacterial levels in people and on surfaces which now underpins a larger-scale study of infection control in hospital environments.
- A collaboration with Public Health England developing mathematical models of the mechanism of antimicrobial resistance (AMR) development in Shigella – a WHO priority AMR organism.
- Modelling of the Nellix endovascular aneurysm sealing (EVAS) system which was instrumental in the decision to withdraw its use from the NHS from Jan 2019.
- Imaging developments in collaboration with Royal Liverpool University Hospital and Walton Neurological Centre, and SMEs Image Analysis Group (London) and Mirada Medical (Oxford), resulting in fast and efficient numerical algorithms for clinical image processing.

LCMH plays an important and successful role in training graduate students and PDRAs. EPSRC iCASE projects with Liverpool Heart and Chest Hospital and Royal Liverpool University Hospital overlapped with this REF period. The student from the former progressed to work with the biomedical department of Astra Zeneca (Cambridge). One PDRA now works on a joint project between the Oxford Oncology Department and Mirada Medical (Oxford), four others hold fellowships at Liverpool, Exeter and Trento (Italy), one a lectureship in the Dept. of Engineering (Liverpool) and one a position with IBM.

LCMH enabled collaborations such as the £2M MRC (Co-I: Bearon, 2017-20) institutional award for skills development, the £800k EU ITN *Renal Toolbox* (Co-I: Bearon, 2018-22), and a £360k grant (PI: Bearon, EP/S033211/1) with the National Biofilms Innovation Centre; Chen is PI on a £1.3M EPSRC collaborative grant on modelling biomechanics of tissues.

CMIT continues to develop and maintain clinical contacts, setting up industrial and challenge-led projects, most recently with medical software companies (YunYing Medical Technology Ltd and Demetrics Medical Technology Ltd) in China.

4.2.2 Mathematical Foundations of Metamaterials: The Waves and Solid Mechanics Group co-lead an EPSRC Programme Grant with Imperial College and Liverpool John Moores on which BAE Systems and the Ministry of Defence are partners and beneficiaries. This has instigated a new theory of electromagnetic and elastic waves in metamaterials at the leading edge of research on dynamic homogenisation, multi-scale analysis of dynamic localization, cloaking and reciprocity of wave phenomena in structured media.

The group participates in CERMAT2, an EU ITN grant funded project joint with the University of Trento (Italy), contributing to research on the dynamics of elastic multi-scale systems and mathematical modelling of fractures and waves in structured solids. The department hosted industrial collaborators from EnginSoft S.p.A., an R&D establishment in Italy with bases in Trento, Padova and Bergamo, which generated impact through the co-creation of software packages for dynamic crack propagation in thermal structured media. They also collaborate with Terumo Aortic (aka Vascutech), a stent design and production company.

4.2.3 Environmental Modelling: The Environmental Radioactivity Research Centre (ERRC) uses radiometric dating to study the effects underpinning environmental and global changes. They collaborate with the Alfred Wegener Institute Helmholtz Centre for Marine & Polar Research, University of Helsinki Environmental Change Research Unit, University of Oslo Department of Geosciences and Lakehead University Department of Sustainability Sciences and Geography, Canada. The ERRC has earned £165k in consultancy fees over the REF period.

In the context of Environmental Safety, A Movchan led a work package for the EU Horizon 2020 Project INDUCE-2-SAFETY (Seismic safety of petrochemical plants), involving ten leading research groups world-wide. In 2020 this will be included in the revision of engineering Eurocodes required for building petrochemical plants in Europe to improve safety.

Participation in major projects benefits the department, and leads to unplanned applications and impact, e.g. Colquitt's work on CERMAT2 led to his developing a new approach to the modelling of earthquakes and the control of full vector elastic wave propagation in active and passive structured solids. This evolved into a project with Menard and the French Forestry Commission

developing and testing a model for spacing boreholes at intervals to create a manmade barrier capable of deflecting an earthquake's shockwave.

4.2.4 Institute for Financial and Actuarial Mathematics: IFAM provides bespoke solutions for company-led problems through co-created research. Its advisory board includes two members from industry. Since 2017, IFAM staff have increased consultancy, earning £59k. A mature example is provided through impact case ICS3 in which start-up Stable Group Ltd uses research by Assa to provide financial security to food-producing and food-purchasing businesses (ICS3). Research on automatic balancing mechanisms in pensions systems (Boado Penas) was recognised in 2020 by first prize in the economics section of Spanish multinational financial service company BBVA.

IFAM has developed strong links with the African Institute of Mathematical Sciences (§4.3.3). They have successfully bid for 4 Overseas Development Agency grants to work on microfinance, microcredit, risk ordering and to fund the attendance of AIMS students and the CEO of the partner microfinance company at the 2019 PARTY workshop in Liverpool. Together with AIMS, and local businesses in Africa, Constantinescu and Menoukeu-Pamen have developed an effective credit scoring system, improving accessibility of lending opportunities for low income individuals. The paper "Stochastic Mortality Modelling for Dependent Coupled Lives", more colloquially `broken-heart syndrome', (Constantinescu, Menoukeu-Pamen, Henshaw, 2020) was based on Ghanaian mortality data collected by AIMS students and resulted in an invitation to write a related article for *The Actuary* magazine.

IFAM runs a summer research internship scheme (S3, S6) for UoL UGs (growing from 8 in 2013 to >50 in 2019, and since 2019 provided as a credit-bearing module for UGs). Students work on research problems provided by industrial partners, typically involving data analysis, with the goal of signalling potential problems and/or managing risk. Industrial partners meet the students at the beginning of the scheme and remain involved throughout, receiving a written report at the end. Fourteen companies have been involved, including Deloitte (consultancy), Satsafe (GPS manufacture), Eddie Stobart (logistics), Innovative Microfinance (Ghanaian microfinance), Moody's Analytics (pensions), Mitsubizhi Bank (finance), Stable (agricultural indexes), and Onward (properties management). This provides valuable experience for our students and has strengthened links with many companies, in one case (Azur, underwriting) leading to a separate paid consultancy.

4.2.5 Other beneficiaries: We encourage individual staff to develop productive relationships with users and support these via the Impact Group (§3.2.3). Karpenkov (Pure) developed an algorithm to improve seasonal trade forecasting for the Russian branch of MediaMarktSaturn, the leading consumer electronics retailer in Europe, saving them an estimated £10M. Sahin (IFAM) runs a separate business providing CPD training for expert witnesses in compensation cases (1786 trained at 104 courses 2015–2020), with collaborators in Turkey. Russell (CEE) introduced a capstone module "Professional Projects and Employability in Mathematics" with Very Group and BAE Systems as project partners in 2019-20. These exemplify the flexible opportunities staff have to achieve impact.

4.3 Indicators of wider influence, contributions to and recognition by the research base

4.3.1 Roles and recognition: Rempe-Gillen was a member of the EPSRC Strategic Advisory Team for Mathematical Sciences (2014-17); Chen, Konstantopoloulos, A Movchan, Newstead (em), and Rees (em) are full EPSRC College members; Bearon, Karpenkov, N Movchan, Pratoussevitch, Sharkey, Thompson and Vasiev are associate members. Bearon and Pratoussevitch are members of the UKRI Future Leaders Fellowship Peer Review College. Parameswaran is a member of the STFC Particle Physics Grants Panel (Theory).

Pratoussevitch sits on the Royal Society's International Exchanges Panel. A Movchan was a member of the ICMS Programme Committee (2002-19). Bearon was elected to the Council of the Institute of Mathematics and its Applications in 2017.

Staff edit international journals across the full spectrum of mathematical sciences, e.g. *Journal of Applied Probability* (Constantinescu), *Quarterly Journal of Mechanics and Applied Mathematics* (A and N Movchan), *Numerical Algorithms* (Chen), *SIAM Multi-Scale Modelling and Simulations* and *Mathematika* (A Movchan), *Arnold Mathematical Journal* (Karpenkov), and *Universe* (Faraggi).

Yuan was awarded the *Ito Prize* (2017) and Karpenkov was a finalist in *SET for Britain* (2015).

4.3.2 Conferences and workshops: During the REF period, staff were lead organiser, co-organiser, or on the local organising committee of 54 conferences and workshops with over 4300 participants. These took place in China, France, Germany, Ghana, Iran, Italy, Japan, New Zealand, Romania, UK and the US; 18 were hosted in Liverpool, and 9 more in the UK. Selected examples are

- SIAM Conference on Control and its Applications (CT19), 2019 in China, 150 participants (Zhang)
- Loops and Legs in Quantum Field Theory, 14th DESY Workshop, 2018, 100 participants (Vogt)
- Stochastic Analysis, Financial and Insurance Mathematics (SAFIM), 2018 in Ghana, 47 participants, (Menoukeu-Pamen)
- 5th Asian Conference on Pattern Recognition (ACPR 2019), 2019 in New Zealand, 100 participants (Chen).
- Resonances of Complex Dynamics, 2018 at the ICMS (Edinburgh), 65 participants (Rempe-Gillen)
- Metamaterials Beyond Photonics, 2016 at the ICMS (Edinburgh), 40 participants (Thompson).

Priority is given to supporting new staff and ECRs to organise workshops and conferences, particularly within Liverpool, which help them to establish their position in a field, e.g.

- The Geometry of Derived Categories, 2019 (69 participants, Rizzardo).
- String Inflation After Planck, 2017 (50 participants, Parameswaran).

The department will host *String Phenomenology* 2022 (postponed from 2021 due to Covid-19); this is the primary international conference in this field attracting 120-150 participants.

Selected plenaries by staff include ones at the *International Conference on Stochastic Methods* (Piunovskiy, 2017), *String Phenomenology* (Faraggi and Parameswaran, 2019), the *Annual International Symposium on Lattice Field Theory* (Schaich, 2019), the *International Congress in Insurance Mathematics and Economics* (Constantinescu, 2017), the *International Symposium on Image Computing and Digital Medicine* (Chen, 2019) and the *International Conference on Vibration Problems* (N Movchan, 2019).

4.3.3 African Institute of Mathematical Sciences: We have built strong links with AIMS, a pan-African network of centres of excellence. In 2016 Menoukeu-Pamen was appointed to a research chair by The Alexander von Humboldt Foundation (retaining 0.2 FTE in Liverpool). This is one of only five such appointments, designed to strengthen mathematical higher education and research in Africa, and promote international networking in mathematics.

The 2018 workshop in Liverpool to kick-start collaboration was attended by AIMS staff from Rwanda, Tanzania, South Africa and Ghana. Subsequently a joint workshop on Data Science was organised at AIMS Ghana with AIMS South Africa participation. UoL staff have spoken at the regular Stochastics and Applications, Research and Training workshops, and the 2018 School on Stochastic Analysis, Financial and Insurance Mathematics in Ghana. These provide platforms for African graduate students and researchers to communicate research findings and share knowledge, and to enhance their abilities to write internationally competitive research proposals.

Constantinescu and Menoukeu-Pamen have supervised 23 MSc students in total from Cameroon, Ghana and Tanzania, and 7 MPhil students from Ghana. Menoukeu-Pamen is primary supervisor for 5 PGRs at AIMS Ghana, and co-supervises two more at Cottbus University, Germany funded by DAAD scholarships. We will strengthen our links with AIMS by increasing exchange activities for staff and students, and have recently (November 2020) signed a memorandum of understanding with them.

4.3.4 Joint PhD programmes: Collaborative PhD programmes with our Chinese partner university XJTLU and Nankai University (China) are springboards for collaborative research projects. In 2019 we signed a 5-year strategic collaboration agreement with Capital Normal University (China) defining a four-year PhD programme with two years at each institution. The memorandum of understanding with AIMS includes provisions for AIMS students to study as PGRs in Liverpool.

We will continue to pursue funding for collaborative projects in Europe with a significant training aspect through whatever mechanisms are available.

4.4 Societal Impact: Engagement with communities and the public

4.4.1 Outreach: The department has a long, proud history of outreach, recognised through the OBE awarded to Prof. Peter Giblin (emeritus) `for services to Mathematics' in 2018. Over the past six years we have widened and deepened this programme initiating a number of innovative activities to increase our public engagement and promote mathematics in creative ways. We enthuse, inspire and create opportunities for developing mathematicians of all ages and from diverse backgrounds. Since September 2013, our sector-leading outreach programme has had 57,600 interactions with pupils and engaged with 950 teachers per year through a rolling

programme of challenges, masterclasses, roadshows, clubs and events, delivered by our dedicated team in conjunction with research staff. We have a special focus on engaging girls and pupils from disadvantaged backgrounds – groups across Merseyside linked with consistently lower STEM uptake than the national average.

Members of the department regularly participate in activities to popularise mathematics, e.g. the evening lecture series “Science in the Snug” and the annual “Liverpool Pint of Science”.

4.4.2 University of Liverpool Mathematics School: In July 2018, the UK Department for Education announced that Liverpool will be the first city in the North of England to host a specialist 16-19 Mathematics School and noted that the “existing university outreach at Liverpool helped with the approval of the maths school”. Lord Agnew was, “pleased that the University of Liverpool has risen to the challenge to give the mathematicians of tomorrow an opportunity to take their talents to the next level”. The school opened in September 2020.

Dr Haslinger (ECR & chess grandmaster) at the UoL Maths School (virtually) in Nov 2020.

4.4.3 Leverhulme Artist in Residence: From February to November 2015 we hosted Leverhulme-funded (PI: Rempe-Gillen) artist in residence, Emily Howard, who composed several pieces inspired by her conversations with staff. One of these, her 2016 BBC Proms Commission `Torus’, was described by The Times as “visionary” and by The Guardian as “one of this year’s finest new works” and won the orchestral category of the 2017 British Composer Awards (see ICS4). Her residency led to departmental involvement in the Royal Northern College of Music’s centre for *Practice and Research in Science and Music* funded by a £1M Research England grant (Bearon & Rempe-Gillen PRiSM Associates).

Prof. Rempe-Gillen presents with Leverhulme artist-in-residence Emily Howard.

4.4.4 Public Lecture Series: In 2017 the department initiated a new public lecture series, the Terry Wall Lectures. The first three speakers were Stanislaw Smirnov (2017), Martin Hairer (2018) and Caucher Birkar (2019), all recent Fields medallists. This new series in pure mathematics complements the well-established Barkla Lecture series in theoretical physics: recent speakers, Katherine Freese (2015), Dame Susan Jocelyn Bell Burnell (2017), 2020 Nobel Prize-winner Roger Penrose (2018) and 2019 Nobel Prize-winner Didier Queloz (2019).

4.5 Contribution to the sustainability of the discipline

Progress in mathematics is driven by its internal dynamic and the stimulus of external problems. As a department we have increased these external stimuli to enrich the mathematics we create. One outcome is the significant numbers of publications relating to material science, economics and insurance, Earth and environmental sciences, biology and medicine, in addition to the those within the more traditional boundaries of mathematics and theoretical physics. These publications arise from the collaborations and partnerships CERMAT2, RARE, INDUCE-2-SAFETY (§4.1, 4.2), and the contributions of our research centres.

To sustain this impetus new generations of researchers must be introduced to these inter-disciplinary areas. We do this is through organising workshops and conferences aimed specifically at PGRs, PDRAs and ECRs in health, metamaterials, financial and actuarial applications, e.g.

- ECR Conference on Data Science, Machine Learning and Artificial Intelligence (with applications to statistics, health and life sciences) co-organised by LCMH, Mathematical Sciences (UoL), Electrical Engineering (UoL), Liverpool Hope University and the Royal Statistical Society (2019)
- Mathematical Modelling in Solids and Structures, CERMAT2 summer school (2015)
- Perspectives on Actuarial Risks in Talks of Young Researchers (PARTY), RARE winter school (2015).

We will maintain and enhance our inclusive environment so that all can thrive and develop as researchers, and contribute to fundamental research and high-impact global challenges. We will encourage collaboration to benefit the mathematical sciences and increase the scope of their influence.