Institution: University of Oxford

Unit of Assessment: 10: Mathematical Sciences

Title of case study: Humanitarian and industrial impacts of mathematical modelling of filtration

Period when the underpinning research was undertaken: 2013–2019

Details of staff conducting the underpinning research from the submitting unit:

Names:	**Roles**	**Periods employed by submitting HEI:**
Chris Breward	Research Fellow, Associate Professor	2001–present
Maria Bruna	Research Fellow	2014–2019
Mohit Dalwadi	PDRA	2014–2015
Ian Griffiths	PDRA, Research Fellow, Professor	2008–present
Sourav Mondal	PDRA	2014–2017
Raka Mondal	PDRA	2015–2016
Colin Please	Professor	2012–present
Galina Printsypar	PDRA	2016–2019

Period when the claimed impact occurred: Aug 2015–Dec 2020

Is this case study continued from a case study submitted in 2014? N

1. Summary of the impact

Mathematical models for the filtration of contaminants flowing through porous media using continuum frameworks have been developed by a team of researchers at the University of Oxford led primarily by Professor Ian Griffiths. This modelling effort has been exploited in four distinct sectors to provide significant societal impact on public health and economic impact through rapid product development.

Firstly, the work has assisted in mitigation of arsenic poisoning in communities living in the Ganges–Brahmaputra Delta, a global hotspot for arsenic groundwater contamination. Using the modelling, researchers at IIT Kharagpur have optimized the use of their inexpensive laterite soil filters and upscaled them from single-home filters to filters for schools and communities. As a result, laterite filters now serve approximately 150,000 people through 130 of the large community-scale filters that have been deployed.

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2. Underpinning research

There have been two main strands in the University of Oxford's work on mathematical modelling of filtration, which began in 2013. The first strand involves the use of asymptotic analysis and homogenization methods to systematically account for the details of the microstructure of the porous filter in models that are posed on the scale of an entire filter device. Traditional mathematical models to describe the behaviour impose unrepresentative restrictions on the regularity of the filter media. Researchers at the University of Oxford, led by Griffiths, have relaxed this constraint in two important ways and introduced models that account for (i) a porosity gradient in the filter [1] and (ii) quasi-periodicity by increasing the size of the unit cell so that 'channel' formation can be considered. This methodology enables the effective diffusivity and permeability of the medium to be determined for filters that are representative of those used by industry [2]. These ideas have been extended to account for trapping of contaminants on the filter surface [3]. Asymptotic methods have also been used to consider catalytic reactions on the surfaces of the porous medium that create liquid that subsequently flows away [4].

The second strand has been to take these macroscopic models and exploit further asymptotic methods to systematically derive simple models that are valid for a specific range of parameter values. In particular, this approach has been very successful in creating an experimentally validated model of arsenic filtration using laterite soil as the porous medium [5]. This latter work has been conducted in collaboration with experimentalists at IIT Kharagpur, India, who discovered the original technology and conducted the experiments to validate the theory. A similar approach has been applied to reduce the complex random nature of particle blocking within a filter to a simple continuum description [6].

3. References to the research

There is a large body of research comprising 28 journal articles that underpin this case study. The following list includes six papers in internationally refereed journals.

[1] M.P. Dalwadi, I.M. Griffiths and M. Bruna. Understanding how porosity gradients can make a better filter using homogenization theory. Proceedings of The Royal Society A. (2015) 471, 20150464. DOI: 10.1098/rspa.2015.0464
[2] G. Printsypar, M. Bruna and I.M. Griffiths. The influence of porous media microstructure on filtration. Journal of Fluid Mechanics. (2019) 861:484–516. DOI: 10.1017/jfm.2018.875
[3] M.P. Dalwadi, M. Bruna and I.M. Griffiths. A multiscale method to calculate filter blockage. Journal of Fluid Mechanics. (2016) 809:264–289. DOI: 10.1017/jfm.2016.656
[4] K.B. Kiradjiev, C.J.W. Breward and I.M. Griffiths. Surface-tension- and injection-driven thin-film flow. Journal of Fluid Mechanics. (2019) 861:765–795 DOI: 10.1017/jfm.2018.934
[5] R. Mondal, S. Mondal, K.V. Kurada, S. Bhattacharjee, S. Sengupta, M. Mondal, S. Karmakar, S. De and I.M. Griffiths. Modelling the transport and adsorption dynamics of arsenic in a soil bed filter. Chemical Engineering Science. (2019) 210:115205. DOI: 10.1016/j.ces.2019.115205
[6] A.U. Krupp, I.M. Griffiths and C.P. Please. Stochastic modelling of membrane filtration. Proceedings of The Royal Society. A (2017) 473, 20160948. DOI: 10.1098/rspa.2016.0948

4. Details of the impact

The filtration models developed by researchers at the University of Oxford have made important contributions to applications in a range of fields where filters are of critical importance. In each case, there was a pre-existing product for which the University of Oxford researchers developed a mathematical model. This model was validated against experimental data, and then used to improve or further develop the product, saving a considerable amount of time and expense compared to existing methods based on extensive experimentation.

4.1 Arsenic removal from groundwater

Background and pathway to impact
The Ganges–Brahmaputra Delta is a global hotspot for arsenic groundwater contamination affecting Indian and Bangladeshi populations. High levels of naturally occurring arsenic, far above the World Health Organization (WHO) safe level of 10µg/L [A], are consumed by people drinking water drawn from shallow wells. This has created a major public health issue in West Bengal and Bangladesh, described by WHO in 2000 [A] as "*the largest mass poisoning of a population in history*". WHO [A] also states: "*it has been estimated that in West Bengal the number of people exposed to arsenic is 1.5 million, and one estimate of the number of patients with arsenicosis exceeds 200,000*", highlighting the urgent need for cost-effective solutions to remove arsenic from water supplies and motivating work on this aspect of the impact.

In 2006, an experimental team led by Prof De from the Chemical Engineering Department at IIT Kharagpur in West Bengal developed a filter for removing arsenic from contaminated water, using naturally abundant laterite soil and flow driven by gravity, and demonstrated that this was ultra-low cost and highly effective at producing water with arsenic below the recommended WHO level [B]. Independently, in 2014, Griffiths began a Royal Society University Research Fellowship project, “21st Century Fluid Dynamical Challenges in Water Purification”. Part of this project concerned mathematical modelling of arsenic removal and he appointed an ex-student of Prof De, Sourav Mondal, as a PDRA in 2015 as part of this work. Griffiths then instigated a collaborative programme between Oxford and IIT Kharagpur, including appointing a second PDRA on a Global Challenges Research Fund grant. Prof De has received several prestigious awards, including national recognition by India in 2018, for the societal benefits of this collaborative work [B,C].

The model of Professor De's filters developed by the University of Oxford team addresses two key requirements: it enables prediction of the lifetime of the filters [D] (beyond which the filter medium becomes saturated with contaminants), and it resolves the question of how they can be upscaled from single home filters to much larger filters for schools and communities. Prof De [B] states “*Ian and his team derived a mathematical model that addressed both of the key requirements. The quantification provided by the mathematical models has allowed us to have confidence as we deploy these filters on a national scale.*” The lifetime prediction, based on the system parameters (flow rate, filter size and arsenic concentration) removed the need for manual inspection, speeding up the deployment of filters in homes, schools and communities as well as increasing the cost-effectiveness of the filter. Prof De [B] indicates that “*this work has enabled us to provide filters with a service life of more than seven years*”.

IIT Kharagpur first licensed the technology to major manufacturers Vas Bros Pvt. Ltd, in 2015, and subsequently Mondal Precision Pvt. Ltd, in 2016, to manufacture these filters [B]. (Note that this Mondal is unrelated to the two PDRAs.) Prof De [B] indicates that “*the mathematical models … have provided additional guidance to Vas Bros and Mondal Precision and confidence to the Government. This filter invention has been approved by the Department of Science & Technology,..., Ministry of Sanitation and Drinking Water* [see [E]], *..., and the West Bengal Arsenic Task force and UNICEF. The number of community scale filters...deployed by our industry partner are about 130 by Mondal Precision Pvt. Ltd and about 12 by Vas Bros till now* [22 Dec 2020]”. The Director of Mondal Precision reports [F] “*We anticipate that this* [Mondal Precision’s 130 filters] *caters to the daily needs of more than 150,000 affected people*”.

Major improvements in public health

The severe health effects of arsenic in the Ganges–Brahmaputra Delta are not well quantified [G]. However, a WHO Report [A] states that “*it has been estimated that the lifetime risk of dying from cancer of the liver, lung, kidney or bladder while drinking 1 litre a day of water containing arsenic at this* [50μg/l] *concentration could be as high as 13 per 1,000 persons exposed*”*.* For the 150,000 people served by the new community-scale filters, this fraction equates to a total of 1,950 premature deaths that have been avoided.

Financial and economic benefits

The upscaled filters are very cheap to run. The Government of India Department of Drinking Water and Sanitation, reporting these laterite filters as a Recommended Technology [E], states this system has a “*lifecycle cost of 31 Rs/Kl* [GBP0.0003 per litre]”). These filters have therefore been financially viable for providing clean water to the affected communities of West Bengal.

There have also been clear consequential improvements in economic activity as a result of fewer early deaths, as well as improvements in general wellbeing and quality of life. Specific savings from these filters are difficult to estimate. However, a Bulletin of WHO [G] states: “*We estimated the economic losses resulting from the arsenic-related mortality burden by calculating lost productivity in terms of per capita gross domestic product (GDP). According to estimates by the International Monetary Fund, the per capita GDP for Bangladesh in 2009 was 1,465 purchasing power parity dollars…could lead to a loss of US$12.5 billion* [USD12,500,000,000]*, provided arsenic exposure (>10 μg/L) remains the same as in 2009*”. Performing a similar calculation for the 1,950 premature deaths avoided in West Bengal, based on the same per capita GDP of USD1,465 per year for deaths avoided, results in gains of 1,950 x USD1,465 per year = USD2,900,000 per year.

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4.5 Conclusion

In summary, the models developed by researchers at the University of Oxford have been used to understand and quantify the behaviour of filtration processes in a multitude of areas and the impact has included: saving many lives, significantly reducing development costs, and dramatically reducing the timeline for new product development.

5. Sources to corroborate the impact

[A] Contamination of drinking-water by arsenic in Bangladesh: a public health emergency, Bulletin of the World Health Organization, vol 78(9) (2000), pp 1093–1103.
[B] Letter from Prof Sirshendu De, Professor of Chemical Engineering and ex-head of department, IIT Kharagpur, India, 22 Dec 2020.
[C] National Meritorious Invention Awards 2018. (award to Prof De)
[D] United Nations University report 'Cost and Efficiency of Arsenic Removal from Groundwater: A Review', 2018.
[E] Government of India Department of Drinking Water and Sanitation, list of recommended technologies for water applications, June 2019.
[F] Letter from the Director of Mondal Precision Private Limited, 4 Oct 2019.
[G] Arsenic in tube well water in Bangladesh: health and economic impacts and implications for arsenic mitigation, Bulletin of the World Health Organization, vol 90(11) (2012), pp 839–846.
[H] [text removed for publication]
[I] [text removed for publication]
[J] Ranking the World's Sulfur Dioxide (SO_2) Hotspots: 2019-2020. Centre for Research on Energy and Clean Air (CREA) Report, 2020.
[K] [text removed for publication]