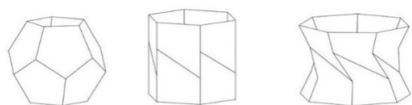


Institution: University of Liverpool		
Unit of Assessment: 10 - Mathematical Sciences		
Title of case study: Pure mathematics informs music to wide critical acclaim and establishes new composer practice		
Period when the underpinning research was undertaken: March 2007 - April 2015		
Details of staff conducting the underpinning research from the submitting unit:		
Name(s):	Role(s) (e.g. job title):	Period(s) employed by submitting HEI:
Prof Lasse Rempe Dr Anna Pratoussevitch	Professor of Pure Mathematics Reader in Pure Mathematics	2006 – present 2006 - present
Period when the claimed impact occurred: February 2015 – November 2020		
Is this case study continued from a case study submitted in 2014? N		
1. Summary of the impact Grounded in Liverpool's dynamical systems research, a pioneering five-year collaboration between Liverpool mathematicians and British composer Emily Howard has generated impacts on creativity, culture and society. Beneficiaries include Howard herself, as well as other composers, audiences (live, radio, streaming), concert venues, publishers, record labels and performing ensembles: • Seven musical works include commissions for BBC Proms and Sir Simon Rattle, live performances at the Royal Albert Hall and the Barbican Centre, as well as five broadcasts on BBC Radio 3. • Concerto for Orchestra hailed by <i>The Times</i> as 'visionary' and recognised with a British Composer Award (2017). • Four published scores, plus two tracks released by record label NMC Recordings. • Lasting influence on artistic practice through a first-of-its-kind centre for science and music (PRiSM).		
2. Underpinning research Lasse Rempe is Professor of Pure Mathematics in the Department of Mathematical Sciences at the University of Liverpool. Rempe researches dynamical systems, specifically the iteration of transcendental functions. In [3.1], with Sebastian van Strien (Imperial College London) he established density of hyperbolicity for the famous Arnold family of circle maps. This family is a simple model for phase-locking in a periodically forced oscillator that has been studied intensively since the 1960s. Density of hyperbolicity is a central question in one-dimensional dynamics (compare Smale's 11th "problem for the next century"). It states, informally, that through a small perturbation within the given family, any system can be changed into one exhibiting the simplest type of behaviour (convergence to stable cycles) – no matter how chaotically the original system behaves. The proof of this result for the Arnold family draws significantly on Rempe's earlier work [3.2] on the escaping set of transcendental entire functions; that is, the set of points where the iterates converge to infinity. In [3.2], Rempe showed that, for a certain class of functions, the behaviour on this set is very rigid - that is, it does not give rise to non-trivial perturbations. The same holds for the complex extension of the Arnold family, with the same proof, and this fact plays a crucial role in [3.1]. This body of research was supported by funding from EPSRC (EP/E017886/1, EP/E052851/1) and the Leverhulme Trust (Philip Leverhulme Prize 2012), totalling GBP529,847. There are four key reasons why Rempe's proof in [3.1] provided fertile ground for developing new creative thinking in his collaboration with composer Emily Howard. Firstly, the result lends		

itself to expression in simple and general terms (no matter how chaotic such a system, there is always stability nearby). Secondly, phase-locking is a phenomenon relevant to musicians – from the synchronisation of linked metronomes to the elimination of non-harmonic overtones through the bowing of a violin string. Thirdly, structure in the complex plane – invisible in the formulation of the problem – plays a crucial part in the proof, which is philosophically intriguing. And, finally, the systems in question could be used to design data series as input into and inspiration for the creative process. From these discussions, Howard evolved new artistic practice underpinned by latest results in dynamical systems research. As an evolution of this creative practice, she went on to draw inspiration directly from other areas of excellent research within Liverpool's Department of Mathematical Sciences. A further example is given below.

Anna Pratoussevitch is a Reader in Pure Mathematics, researching **geometry and topology**. To explore pseudo-Riemannian space forms of constant curvature, Pratoussevitch constructs fundamental domains in the shape of higher-dimensional polyhedra [3.3, 3.4]. A central question of low-dimensional topology has been the classification of 3-dimensional manifolds. Thurston's approach to this problem, that any 3-manifold can be constructed from building blocks carrying one of 8 Riemannian geometries, has contributed significantly to our understanding of low-dimensional manifolds, including the celebrated proof of the Poincaré conjecture by Perelman. Pratoussevitch's work explores the implications of this approach in pseudo-Riemannian 3-manifolds by describing how pseudo-Riemannian manifolds of constant curvature can be constructed from pseudo-Riemannian polyhedra by gluing together pairs of 2-dimensional faces. The constructions of building block manifolds from polyhedra in both Riemannian and pseudo-Riemannian geometry give excellent visual representations of 3-dimensional objects that cannot be easily visualized in 3D space. Pratoussevitch explored these fundamental ideas of positive and negative curvature in different spaces alongside images of the polyhedra with composer Howard, initiating new creative thinking about shapes and space to inspire new orchestral music.



Examples of polyhedra studied by Pratoussevitch that influenced geometry-inspired orchestral works.

3. References to the research

3.1. Rempe-Gillen L and Van Strien S. (2015) Density of hyperbolicity for classes of real transcendental entire functions and circle maps. *Duke Math. J.*, 164(6): 1079-1137. [doi:10.1215/00127094-2885764](https://doi.org/10.1215/00127094-2885764)

3.2. Rempe L. (2009). Rigidity of escaping dynamics for transcendental entire functions. *Acta Mathematica*, 203(2), 235-267. [doi:10.1007/s11511-009-0042-y](https://doi.org/10.1007/s11511-009-0042-y)

3.3. Pratoussevitch A. (2007) Fundamental domains in Lorentzian geometry. *Geom. Dedicata* 126: 155-175. [doi:10.1007/s10711-006-9117-5](https://doi.org/10.1007/s10711-006-9117-5)

3.4. Pratoussevitch A (2011) The Combinatorial Geometry of Q-Gorenstein Quasi-Homogeneous Surface Singularities. *Diff. Geometry and its Appl.* 29: 507-515. [doi:10.1016/j.difgeo.2011.04.031](https://doi.org/10.1016/j.difgeo.2011.04.031)

4. Details of the impact

This body of pure mathematics research has brought about three principal impacts on creativity, culture and society:

- **4.1.** New creative thinking that transformed the practice and career of composer Emily Howard [5.1, 5.2].
- **4.2.** Seven musical works over five years, with diverse coverage to wide critical acclaim [5.1, 5.3-8].
- **4.3.** Dedicated centre for Practice & Research in Science & Music (PRiSM) in Manchester that is influencing composer practice nationally [5.1, 5.8-10].

4.1. Dynamical systems research transforms practice of composer

Rempe and Liverpool's Department of Mathematical Sciences hosted freelance composer Emily Howard as Leverhulme Trust Artist in Residence between February and November 2015 [5.2]. A mathematics graduate herself, Howard's fluency in mathematical language enabled her not only to absorb general ideas, but also to discuss details and subtleties of contemporary mathematical research. According to Howard, this made for an *'appealing and distinctive'* partnership [5.1].

Working with Rempe and Pratoussevitch provided Howard with unique insights into frontier mathematical research, taking her creative ideas beyond classical mathematical principles for the first time [5.1]. Howard joined Rempe and his group in discussions about their research throughout 2015, with particular focus on Rempe's work on the Arnold family [3.1]. The group created two sets of numeric data for Howard, encapsulating key ideas in the result and its proof. These datasets and discussions were pivotal creative input for Howard's 2015 compositions described below, establishing enduring principles for her entire approach to composition [5.1]. Howard explains how this opportunity *'transformed my creative practice at the time and has continued to influence my entire creative approach and musical works since'* [5.1].

4.2. Audiences, listeners, music venues, publishers, record labels and performing ensembles benefit from new critically-acclaimed music

The body of research [3.1-4] influenced seven critically acclaimed musical works of increasing profile [5.3-4]. Audiences in the UK, Germany, Canada and the US, BBC audiences and the wider public have benefitted from sold-out live performances, radio and streaming [5.3-4]. Edition Peters, a world-leading publisher of classical sheet music who only publishes music with *'sufficient commercial benefit'* [5.5a], signed Howard in March 2019 [5.5b]. Accompanying programme notes detail how the music is a response to Liverpool's research [5.5c].

Torus (2016) is the first of three geometry-inspired orchestral works drawing on Howard's engagement with mathematics research. *Torus* was a **BBC Proms** co-commission with the Royal Liverpool Philharmonic Orchestra, first performed in 2016 at the iconic **Royal Albert Hall** [5.3a] to a sell-out audience of 4,294 [5.3b]. Hailed by *The Guardian* as *'one of this year's finest new works'* [5.6a] and *'visionary'* by *The Times* [5.6b], the piece was recognised with a 2017 **British Composer Award** [5.6c], the highest industry recognition for excellence in classical composition. *Torus* was described by the judges as *'captivating from start to finish'* [5.6c].

The **Barbican Centre, London**, Europe's largest performing arts centre, where a live performance of *Torus* by the BBC Symphony Orchestra took place in 2019, describe the piece as *'startling'* [5.3c, 5.8]. *Torus* has featured twice on BBC Radio 3 [5.3d,e], most recently in 2019 to approximately 333,000 listeners [5.3f]. Howard deliberately included in *Torus* a *'crucial "perturbation": a moment where, as in your [Rempe's] work, the parameters of the system change slightly to dramatic effect'* [5.1]. *sphere* (2017) was commissioned by the Bamberg Symphony Orchestra, Germany and broadcast on **BBC Radio 3** [5.3g] to approximately 310,000 listeners [5.3f]. Howard explains how *'Several conceptual ideas from dynamical systems [Rempe] as well as ideas from topology and geometry [Pratoussevitch] stemming from conversations with researchers at Liverpool have noticeably influenced the work'* [5.2].

Antisphere was commissioned by the Barbican with internationally renowned conductor **Sir Simon Rattle** to open the **London Symphony Orchestra's** 2019/20 season [5.3h, 5.8] to a live audience of 1,419 people [5.3i]. The Barbican's Classical Programming Coordinator describes this *'dynamic contribution'* to the Barbican's 2019/20 programme - *'bringing to life the close connections between the arts and sciences in a truly engaging way'* and bringing about *'widespread media coverage in national outlets, including interviews on BBC Radio 4'* [5.8]. By this time, Howard was credited by Sir Simon Rattle as a *'gifted composer'*, with her music always having *'a scientific picture behind it'* and her *'own very particular sound world'* [5.7].

Other pieces influenced directly by Rempe's research include *Leviathan* (2015), creating 'a newly-formed and unpredictable musical language, which responds to recent mathematical research in dynamical systems' [5.5c]. Howard explains why this research was important: 'The contrast of chaos in a...system and the stability of neighbouring systems was of particular interest to me, and the connection to physical systems – in particular phase-locking – and its relevance to musical instruments' [5.1]. *Leviathan* was performed in 2015 in Montreal, Toronto, Detroit, Chicago and New York [5.5c] as well as six venues in the UK.

Howard's song *Threnos* (2015) was broadcast by **BBC Singers** in November 2019 [5.3j] to approximately 227,000 radio listeners [5.3f]. *Threnos* and *Leviathan* were recorded for the **NMC Recordings Label** and commercially released in 2016 on Howard's NMC Disc *Magnetite* [5.4], which received four stars from **The Guardian** [5.6d]. The album liner explains how dynamical systems (specifically the chaotic motion of a forced oscillator) directly informed the time structure of the work [5.4d]. At November 2020 it had been streamed 32,950 times, more than ten times that of an equivalent collection released that year [5.4e].

Orbit 1a and *Orbit 2a* (2015) are a direct response to the datasets provided by Rempe's group. Howard explains 'I selected different musical properties to represent unstable and stable periodic orbits and depicted the specific moment of perturbation (where the underlying rules of the systems change) as a definite event' [5.1]. Continuing the Orbits series, *Chaos or Chess* (2016) is influenced in a 'more abstract conceptual way by your [Rempe's] proof and our stimulating discussions', as for *Threnos* [5.1]. *Orbit 1a* and *Chaos or Chess* were performed live on **BBC Radio 3** in 2016 [5.3k] and the latter at the Edinburgh Science Festival in 2019 [5.3l].

4.3. New dedicated centre for science and music (PRiSM) influences other composers

Engaging with Liverpool's research [3.1-4] transformed Howard's approach to composition, shifting the foundations of her entire creative practice, career and outputs: 'Since we [Howard and Rempe] pioneered this approach, I actively seek out other mathematicians and scientists as rich sources of creative input, made possible through the mathematical confidence I developed during the Liverpool residency...Through the vehicle of **PRiSM [centre for Practice & Research in Science & Music]** we are influencing other composers to change creative practice at national scale - bringing together a unique community of practitioners and researchers. This propagation is evidence of the success and sustainability of the creative approach we pioneered together in Liverpool' [5.1].

In 2017 Howard co-established PRiSM at the Royal Northern College of Music, Manchester, of which Rempe is a founding Associate member [5.1, 5.9a]. **Marcus du Sautoy** (Co-Director of PRiSM and Simonyi Professor, University of Oxford) asserts Rempe's 'pioneering collaboration with Emily was instrumental to the inception of PRiSM' [5.10].

PRiSM is building on the approach developed at Liverpool to benefit composers, curators and audiences through high-profile commissions. Artificial Intelligence (AI)-generated music in association with the BBC Philharmonic Orchestra [5.9b] is one example. This featured on the **Royal Institution Christmas Lectures 2019** [5.9c], broadcast on BBC 4 and available afterwards on BBC iPlayer, attracting 559,813 views (03-03-2020) [5.9d]. Another composer explains how a PRiSM collaboration in 2020 changed her practice: 'I had never before used a scientific paper as a starting point for creating music, and now I am looking at using many different types of text to inform my work as an opera composer' [5.9e]. The **Barbican Centre** evaluates a PRiSM event in 2019: a 'ground-breaking team of composers from PRiSM...leading to new insights', enabling the venue to 'reach an audience outside of our usual classical attenders', bringing 'almost double the expected audience numbers'; it 'worked so well ...we are also looking for further opportunities to involve guest curators in our work'; benefits to performers include 'using new technologies... which they intend to use more in their day-to-day work' [5.8].

'With the world around us changing, I believe the Barbican's vision of 'Arts without Boundaries' is more vital than ever and that Emily Howard and PRiSM's innovative works are advancing our progress in realising this vision' [5.8] – The Barbican Centre.

5. Sources to corroborate the impact

5.1. Letter from composer Emily Howard explaining how Liverpool's pure mathematics research transformed her practice, resulting in high-profile musical works and establishment of PRiSM.

5.2. Leverhulme Trust Artist in Residence final report (2016). Howard and Rempe (p.3 and 7).

5.3. Event billings of live and radio performances and associated audience figures. **a)** [BBC Proms 2016, Prom 53. Emily Howard: Torus \(25 August 2016, Royal Albert Hall\)](#). **b)** Email from BBC Proms Marketing Manager confirming full capacity audience 4,294 for a). **c)** [BBC Symphony Orchestra & Chorus 2019-20 Season. Emily Howard: Torus \(1 November 2019, Barbican, London\)](#). **d)** [BBC Proms 2016, Prom 53. Emily Howard: Torus \(25 August 2016, BBC Radio 3\)](#). **e)** [BBC Radio 3, Afternoon Concert. Emily Howard: Torus \(11 November 2019\)](#). **f)** Email from BBC Senior Research Executive confirming listener figures for e), g) and j). **g)** [BBC Radio 3, Afternoon Concert. Emily Howard: sphere \(2 July 2018\)](#). **h)** [London Symphony Orchestra/Sir Simon Rattle. Emily Howard: Antisphere \(14 September 2019, Barbican, London\)](#). **i)** Email from the Barbican's Classical Programming Coordinator confirming audience figure for h). **j)** [BBC Radio 3, Afternoon Concert. Emily Howard: Threnos \(15 November 2019\)](#). **k)** [BBC Radio 3. Proms Extra \(25 August 2016\). Composers in Conversation: Emily Howard. Featuring live performances of Orbit 1a and Chaos or Chess](#). **l)** An afternoon of music and mathematics. (20 April 2019) [Edinburgh Science Festival 2019 programme](#) (p.56).

5.4. NMC Recording Label Debut Disc Magnetite (2016) Howard, E. Features pieces *Leviathan* and *Threnos*. **a)** *Magnetite* album on sale from [Amazon](#) and available to stream on Amazon Music. **b)** *Magnetite* album available to stream on [Apple Music](#). **c)** *Magnetite* album available to stream on [Spotify](#). **d)** *Magnetite* album liner (pdf) explains link to dynamical systems research (see *Leviathan*, pp.7-8). **e)** Email from the General Manager, NMC Recordings with comparative streaming figures for the *Magnetite* album (November 2020).

5.5. Classical music publisher [Edition Peters](#) website. **a)** Statement that Edition Peters accepts only music with '[sufficient commercial benefit](#)'. **b)** Announcement of [signing of Howard](#) (March 2019). **c)** [Full musical scores](#) for *Leviathan*; *Torus*; *Threnos*; *sphere* on public sale; *Leviathan* programme notes refer to dynamical systems research and North American Tour.

5.6. Examples of media reviews of Emily Howard's music. **a)** *Torus* - [Our critics' proms highlights - what were yours?](#) (12 September 2016). The Guardian. (See 'George Hall' section). **b)** *Torus - Concert: Prom 53: RLPO/ Petrenko at the Royal Albert Hall* (29 August 2016). The Times. (See subheading '*In the visionary Torus...*'). **c)** *Torus - British Composer Awards 2017 Winners Announced (7 Dec 2017)*. PRS for Music. **d)** [Emily Howard: Magnetite CD review – delicate, detailed and rigorous](#) (21 August 2016). The Guardian.

5.7. [Video interview](#) with conductor Sir Simon Rattle (September 2019). See 1m 53s–3m.

5.8. Letter from the Barbican Centre's Classical Programming Coordinator on *Torus*, *Antisphere* and a PRiSM commission, including associated impact and their professional opinion.

5.9. [Centre for Practice & Research in Science & Music \(PRiSM\)](#) activity. **a)** [PRiSM launch](#) in October 2017 (with Rempe). **b)** Announcement of [PRiSM composer in association with the BBC Philharmonic Orchestra](#), July 2018. **c)** Royal Institution Christmas Lectures 2019, [How Can We All Win?](#) (28 December 2019) featuring AI-generated music by PRiSM composer (see 43-47m, plus 57m–59m 5s, including acknowledgement in closing credits at 58m 42s). **d)** Broadcasters Audience Research Board (BARB) viewing figures for c), retrieved 3rd March 2020. **e)** Email from opera composer on how PRiSM has influenced her practice (November 2020).

5.10. Letter from Professor Marcus du Sautoy, Co-Director of PRiSM and Charles Simonyi Professor for the Public Understanding of Science, University of Oxford, explaining Rempe's contribution to the inception of PRiSM.