Institution: University of Bristol		
Unit of Assessment: 23) Education		
Title of case study: Embedding "mastery" teaching in primary mathematics		
Period when the underpinning research was undertaken: 2014 - 2020		
Details of staff conducting the underpinning research from the submitting unit:		
Name(s): Alf Coles	**Role(s) (e.g. job title):** Professor of Mathematics Education	**Period(s) employed by submitting HEI:** 2010 - present
Period when the claimed impact occurred: 2017 – 2020		
Is this case study continued from a case study submitted in 2014? No		

1. Summary of the impact

"Mastery" teaching in mathematics is a UK government initiative with the ambition to eradicate under-achievement. University of Bristol research has shaped the UK's National Centre for Excellence in Teaching Mathematics (NCETM) Professional Development materials and resources for primary schools which have become the definitive interpretation of mastery teaching in England. The materials are backed by the Department for Education (DfE) and heavily influenced the new DfE/NCETM Primary Mathematics National Curriculum guidance. The materials are in use by a network of 40 Maths Hubs and 8,000 primary schools in England, with preliminary evidence of improvement in student understanding and attainment. The approach also informed primary schools maths textbooks in Mexico used by 4.5 million children.

2. Underpinning research

Under-achievement of students in mathematics is a fundamental problem in the UK. In England, a stubborn 20% of each cohort, by age 15, attain levels of numeracy judged internationally not to be suitable for life in a modern society (Level 1 or below on PISA tests). We know that social mobility is linked directly to attainment in mathematics, with the subject acting as gatekeeper for access to Higher Education and professional qualification. Coles' research, in collaboration with Sinclair (Simon Fraser University Vancouver), has showed that one reason for this under-achievement is that mathematical concepts can be presented to students in incoherent ways or in ways that do not match well with children's lived experiences. Their research calls into question prevalent assumptions about curriculum organisation and teaching approaches, both within the academic community and current school practices. There are three strands to their research findings and conceptual arguments:

What is a number?

Children's first engagements with number in the early years of primary school tend to be in contexts of answering "how many?" questions. Yet outside school, it is clear that meaningful experiences include, for example, engaging in number as an ordered sequence (e.g. reciting "one-two-three…", etc) in which quantity is less important than how one number relates to its neighbours. Coles and Sinclair have conducted empirical studies which indicate that ways in which children operate with numbers are often beyond curriculum expectations, raising concerns that the expectation of linear progression can lead to lack of exposure to more sophisticated techniques [2]. Their research makes the case for offering children a balance of exposure to number as a relation or measure, and number as linked to objects or quantity [5, 6].

The role of place value

A significant barrier to children's progress in mathematics occurs if they cannot interpret the written form of two-digit numbers (e.g. 23) to indicate two tens and three units. Typically work on place value makes use of materials that emphasises number as object and quantity. Empirical work carried out by Coles and Sinclair has highlighted alternative and powerful routes to becoming successful with place value conventions that draw on children's facility with language and gesture – building on how numbers relate to one another in a naming structure, more than how numbers relate to objects. For instance, they have demonstrated the benefits of working with a "dual naming" of the numbers 11-99, that makes the place value component of these numbers linguistically explicit (e.g., "23" has the dual names of "twenty-three" and "two-ty three") [4, 6]. Their research has helped to de-mystify what is involved in understanding place value, for instance, that the "place" of a number indicates its value only in a number's written form, whereas in spoken language, it is labels (such as "hundred") that indicate value (and e.g., phrases such as "four and twenty" are meaningful, where the standard place is swapped).

Developmental assumptions

Coles and Sinclair have brought into question assumptions, prevalent across many countries, that learning in mathematics must proceed from operations on concrete objects, into work with abstract and dis-embodied concepts [3]. Their critique suggests, for example, that there is no basis for current practices of excluding some students from "abstract" mathematics on the basis that they are not ready for it. They show how choices of tool use and ways of working can disrupt typical orders in which students learn particular concepts [2] such that, for instance, student facility with what is usually taken to be 'concrete' and 'abstract' might take place simultaneously. Their research demonstrates how uses of particular representations and digital tools prompts students' creative use of number concepts and their mathematical reasoning [1]. They have shown how children, across a range of levels of prior attainment, can be supported to reason about and discuss mathematical ideas that would usually be seen as beyond them [6] *if* the mathematics is offered and represented in a structured and coherent manner [1]. Their research proposes a novel conception of how children come to be successful at mathematics, avoiding past dualist separations of thought and action.

3. References to the research

[1] **Coles A**, Sinclair N. 2019, Ritualisation in early number work. *Educational Studies in Mathematics*, 101, 177-194. DOI:10.1007/s10649-018-9815-6 (2,357 downloads, 10.03.20)

[2] **Coles A,** Sinclair N. 2018, Re-thinking 'normal' development in the early learning of number, *J Numerical Cognition*, 4, 136-158. DOI:10.5964/jnc.v4i1.101 (942 downloads, 10.03.20)

[3] **Coles A,** Sinclair N. 2019. Re-thinking 'Concrete to Abstract' in Mathematics Education: Towards the Use of Symbolically Structured Environments. *Canadian J Science, Mathematics and Technology Education*. 19, 465–480. DOI:10.1007/s42330-019-00068-4

[4] **Coles A**, & Sinclair N. 2017, Re-thinking place value: from metaphor to metonym. *For the Learning of Mathematics,* 37, 3-8.

[5] Sinclair N, & **Coles A**. 2017, Returning to ordinality in early number sense: Neurological, technological and pedagogical considerations. In F. Ferrara, E. Faggiano, A. Montone (Eds.) Innovation and Technologies in Mathematics Education. Springer: Rotterdam, pp.39-58.

[6] **Coles A**. 2014, Transitional devices. *For the Learning of Mathematics,* 34, 24-30.

Research Funding:

[i] **Coles A**, Teaching and learning primary mathematics, ESRC Impact Acceleration Account Exchange, 2017 – 2018, GBP23,000

4. Details of the impact

Coles' research has directly informed education policy in England. This has led to improvements in mathematical teaching practices in English primary schools, which in turn is having a positive impact on student outcomes.

Informing education policy in England

Coles' research shaped the development of the National Centre for Excellence in the Teaching of Mathematics (NCETM) Primary Mastery Professional Development (PD) materials [A, B], which "*are supported by the Department for Education and will be disseminated via the 35 Maths Hub across England"* [B]. These resources then *'heavily influenced'* the new NCETM and Department for Education (DfE) Primary Mathematics National Curriculum Guidance Document (2020) [I, J], as set out below. The research has made a distinct and material contribution to the content of the professional development materials and curriculum guidance in three ways:

A broader conception of number: following directly from Coles and Sinclair's arguments [1, 2 & 5], there is an explicit balance between different concepts of number in the materials. In particular, the PD materials [A] are explicit that teachers should be offering students experiences of "number as object" (a concept) at the same time as experiences of "number as length" (a more relational concept) [2, 5]. For instance, the very first teaching point of the Addition and Subtraction "spine" addresses number as a measure, *before* introducing number as linked to objects (e.g. see "Spine 1 overview" in [A]; National Curriculum Guidance [I], pp.20-23, work on numbers 1-20 as location and length, before object), a reversal of the historically typical order.

Downplaying place value: whereas the National Curriculum has a separate strand for "Place Value" within learning number in primary school, in the NCETM PD materials there is no separate "place value" strand [4] (the three current "spines" [A] subsume all National Curriculum targets for place value). Work on place value is embedded within strands of other number work, serving to downplay its importance, as called for by Coles and Sinclair [4]. Their proposals for a "dual naming" of numbers 11-99 [6] has been written into the materials and guidance now going to all primary schools (Spine 1, Section 1.9 [A]). In line with their recommendations, the order of the NCETM materials (Spine 1, Section 1.9 and 1.10) mean that children will now work on the numbers 20-100, which are mainly named regularly (e.g. "43" is said "four"/"ty"/"three"; "95" is said "nine"/"ty"/"five") before coming back to work on the irregularly named numbers from 10-19 (also see National Curriculum Guidance [I], pp.18-19).

Use of resources and representations: one of the resources proposed by Coles and Sinclair is the "Gattegno Tens Chart", as a device for allowing both a linguistic approach to place value and a device that allows teachers and students to work directly on 'abstract' elements of number structure [1, 6]. In the NCETM materials there is significant use of the "Gattegno Tens Chart" in the materials [1, 6], (e.g. see Spine 1, Section 1.8, Teaching point 3, from [A]; and National Curriculum Guidance [I], p.19). The chart is also explicitly mentioned as one of the core representations of number that should be used consistently throughout the primary years and extending into the first years of secondary school.

Improved maths pedagogy of primary teachers in England

All the NCETM materials are freely available on their website, with approximately 24,000 downloads per month (comprising approximately 100,000 individual files) [Hi], and over 8,000 primary schools engaged with the mastery programme [J]. Coles has run professional

development activities supporting teachers to understand the basis of the new approaches to learning mathematics, for example keynote talks at conferences and conference sessions (Jurassic Maths Hub conference, June 2020; Cabot Learning Federation Mathematics Meeting, March 2018; South West Primary Maths Conference, November 2017; Boolean Hub Conference, January 2017) [F]. The NCETM materials are not statutory and the level of take-up by schools (over half of all primary schools) [J], is evidence of their relevance to teachers.

There is evidence of primary teachers making use of the materials, including some who were sceptical of their value but are *"now their biggest fans"* and can see *"how confident the Year 1s were in their maths talking and reasoning"* [Ci, Cii]. In a recent evaluation involving 5 schools the impact "…*on teachers was felt particularly on their subject knowledge, confidence, and the precision of their teaching. In these schools there was evidence that students are behaving in ways that have not been seen before, in terms of their enjoyment of mathematics, their confidence in the subject, … and in terms of their ability to use reasoning and mathematical language*." [Ci]. An ESRC Impact Acceleration Award [i] led to a report by Coles on one 'early adopter' school's experiences with the materials. Positive impacts were found on teachers in the school and there was evidence of changes in teaching practice, linked to a professional cycle using the materials, trialled through the ESRC IAA project [i]. This evaluation evidence was also reported in the NCETM Newsletter [D (p.9-16)] and sent to all primary schools in England.

The NCETM now has 40 Maths Hubs [Hiii] and in each one there is a group of teachers from six or seven different schools working on primary mastery in "Teaching for Mastery Work Groups", including the use and implementation of the materials (overall, therefore, involving teachers in approximately 240 schools) [Hii]. Testimony of teachers running these groups [Hii, J] points to impact on teacher knowledge, teaching practice and resulting increase of confidence of students in mathematics. The Director of the NCETM observed: *""Feedback from teachers about the materials has been overwhelmingly positive, suggesting that the materials have helped to improve teachers' mathematical subject knowledge, as well as improving their pedagogical subject knowledge and supporting them to developing their pedagogical skills*." [J]. These impacts are corroborated by a report from the NCETM, conducted independently, which summarised interviews with mathematics leads in five primary schools [Ci]. Themes in this report include changes in teachers' understanding of the power of representations to become tools for children's thinking, rather than supports for algorithmic procedures: "*We have always used the base 10 resources alongside a pictorial representation but in previous years that felt more like the method rather than the concept. Now it feels like [the representation] is there to support but lots of them are doing it mentally. I am not saying the 'win' is doing it mentally but being able to step back from it"* [Ci]. The teachers across the schools spoke about the PD materials changing their approach to teaching mathematics and their own understanding of number: *"Teacher subject knowledge has definitely improved, especially for teachers who are early in their career. I think they have got a much better understanding of the structure of addition, subtraction, multiplication and division; I think that is the biggest thing, that they are understanding"* [Ci].

Further national impact comes through Coles' work with the professional body the Association of Teachers of Mathematics, for whose magazine he writes regularly [E] and for whom he has run research presentations, at local and national conferences, both prior to and since 2014 (e.g. February 2015, Birmingham ATM branch [60 teachers]; March 2011, ATM plenary speaker [450 attendees]), influencing mathematics teachers and their approaches.

Increased pupil mastery in mathematics

Emerging evidence reveals the direct benefits to the students whose experience of learning mathematics is conceptually coherent, supporting attainment and opportunities for mathematical reasoning [C, H]. Teachers in the NCETM report [Ci] spoke about never before having seen students understand and reason about mathematics in the way that they did when the PD materials were used as the basis for lesson planning:

"I have taught the same things for the past five years and this is the only year I can actually say they have really understood fractions thanks to the division and multiplication that went before it"

"...you realise that if you give them the language structure then they can reason, if you don't what hope have they got, but the materials offer that structure, and you have all these visuals that you can compare and talk about"

"...blown away by how [students] were able to discuss mathematics at the end of year 1 in a way [we] had not seen a year 1 class do before".

Informing international education policy and practice

The overall approach to teaching and learning mathematics has also influenced the development of textbooks in Mexico and in particular the Teacher Guides which, along with the national textbooks, are mandated by the Mexican government for use in all schools, thereby shaping mathematics teaching and learning strategies that reach 4.5 million primary school children [G]. The textbook author noted Coles' work *"on the development of classroom cultures and on learning mathematics (and early number) led to advice to teachers to promote cultures in which students explore mathematical structure and where making mistakes is part of doing (and therefore learning) mathematics)"* [G].

5. Sources to corroborate the impact

[A] NCETM (2017-2019). Primary Mastery Professional Development Materials Year 1 to 6:
Spine 1: Number, Addition and Subtraction (2017)
Spine 2: Multiplication and Division (2019)
Spine 3: Fractions (2019)
Coles is listed on these pages as the sole educational consultant for the materials

[B] NCETM (2020). Factual statement - Director for Primary

[C] (i) Helme R. (2020). Teaching for mastery: A report on the impact of the use of the NCETM Primary Mastery Professional Development materials in 5 schools
(ii) Bristol Cathedral Primary School. (2019). Factual statement

[D] NCETM (2018). One school's experience of using the NCETM's mastery materials for the first time, Primary & Early Years Magazine 105, p.9-16.

[E] Association of Teachers of Maths (ATM) (2016). ATM Newsletter: Everyone can be a mathematician

[F] South West Primary Maths conference: It's good to talk (2017). Feedback report

[G] UDLAP (2020). Factual statement - Mexico Primary School text-book author

[H] NCETM (2019). (i) Email correspondence (16/11/20) - download statistics (ii) Web page: Primary Teaching for Mastery Development Work Groups (iii) Web page: Find Your Hub

[I] DfE (2020). Teaching mathematics in primary schools Website - Mathematics guidance: key stages 1 and 2

[J] NCETM (2020). Factual statement - Director of NCETM