Institution: University of Birmingham		
Unit of Assessment: UoA10, Mathematics		
Title of case study: Nonlinear semidefinite programming optimises and stabilises risk calculations, saving an estimated €57 million for the world's 5th largest insurance company		
Period when the underpinning research was undertaken: 2009–2010		
Details of staff conducting the underpinning research from the submitting unit:		
Name(s):	**Role(s) (e.g. job title):**	**Period(s) employed by submitting HEI:**
Prof. Michal Kočvara	Professor in Mathematical Optimisation	January 2007–present
Period when the claimed impact occurred: 2016–present		
Is this case study continued from a case study submitted in 2014? No		

1. Summary of the impact

The impact of this case study is **commercial**. University of Birmingham (UoB) research into a novel nonlinear semidefinite programming (NLP-SDP) formulation of the nearest correlation matrix problem led to **the development of a new technology**, the bespoke optimisation software package PENCOR.

This **new technology** was **commercially adopted** by the world's 5th largest insurance business, Allianz SE. Allianz is a **global company**, actively managing around **€764 billion in assets for its customers** and operating as an international insurer in **79 countries**. Since 2016, PENCOR has **improved the performance** of Allianz in two key ways:

- **Improved methods of risk assessment leading to regulatory compliance**: PENCOR is used as a critical component in ensuring compliance with the Solvency II regulatory framework, a legal requirement for continued trading for all insurers headquartered in the EU since 2016.
- **Significant gains in productivity and improved cost-effectiveness**: PENCOR has avoided lost profits for Allianz of an estimated €57 million since 2016.

2. Underpinning research

Our impact is underpinned by the creation of an optimisation methodology for quantifying risk that Prof. Michal Kočvara subsequently developed into a bespoke optimisation software product, PENCOR, solving a long-standing problem in mathematical finance. The development of PENCOR and its direct relevance to the finance sector are outlined in the following four stages:

1) Background: real-world scenario generation in finance

When making investments, financial organisations calculate the risk associated with their portfolio. A critical step in this calculation entails running a very large number of (economic) scenarios, which represent future states of the world. Careful scenario selection and accurate profit/loss computation is required annually by law for all insurers headquartered in the EU, as part of the Solvency II regulatory framework.

Crucially, constructing these scenarios requires that the correlation of thousands of risk factors is modelled in a proper and reliable way. These correlations are summarised into a very large correlation matrix.

2) <u>The problem: the Nearest Correlation Matrix (NCM) in economic prediction</u>

A correlation matrix is a symmetric positive semidefinite matrix with unit diagonal. When constructing correlation matrices for economic scenario predictions, the real-world data from which the correlations are constructed is often asynchronous or incomplete, or the models themselves may need to be stress-tested by artificially adjusting individual correlations. These factors result in an approximate correlation matrix (ACM) that is unlikely to be positive semidefinite. Subsequent risk calculations using such an approximate matrix may break down altogether, for instance due to negative volatilities. As such, it is necessary from a practical standpoint to compute the nearest correlation matrix (NCM) to the ACM, and then use the NCM in subsequent market analyses.

The ACM for real-world scenario generation is dense, usually with a dimension in the thousands. A feature of the ACM is that relatively few vectors of observations are available, and so the ACM has low rank. This usually leads to an NCM that is very poorly conditioned, i.e. the outputs from scenario generation are highly unstable, with large changes in solution resulting from small changes in inputs. This instability is highly undesirable, both from the point of view of quantifying market risk and satisfying Solvency II regulations.

3) <u>The solution: bypassing problems with the NCM via a Nonlinear Semidefinite Programme Formulation</u>

In order to resolve the issue of poorly conditioned NCMs, Kočvara **reformulated the NCM problem by introducing a constraint on the matrix condition number**. The resulting NCM problem is a more complicated nonlinear semidefinite problem, which can be solved via application of the body of algorithmic developments that Kočvara's group have made in nonlinear semidefinite programming [1]. The resulting NCM, which has a controlled condition number, then provides very stable predictions in the risk analysis. This **key finding** represents a step-change in semidefinite programming in financial optimisation.

4) <u>The application: solving NLP-SDP with PENCOR</u>

Kočvara, Stingl (Erlangen) and Fiala (UoB, Numerical Algorithms Group) developed a mathematical algorithm and an associated software PENNON for solving linear semidefinite programming (SDP) problems, formulations with bilinear matrix inequalities (BMI), and NLP-SDP problems. Using the key research finding — the NLP-SDP formulation of the NCM problem [1] — as a basis, Kočvara led the development of a new product based on PENNON. Known as **PENCOR**, this software was developed specifically for the Group Risk Division of Allianz SE [2]. Kočvara provided the underpinning research, the theoretical formulation of the problem as an NLP-SDP [1], while Stingl led the implementation of these problems on a multicore architecture

3. References to the research

1. M. Kočvara and M. Stingl. PENNON: Software for Linear and Nonlinear Matrix Inequalities. In: *Handbook on Semidefinite, Conic and Polynomial Optimization*, Miguel F. Anjos and Jean B. Lasserre (eds), Springer, 2012, pp. 755–794. ISBN 978-1-4614-0768-3

2. M. Kočvara and M. Stingl. PENCOR-MT User's Guide. A technical report compiled for Allianz, December 2010.

4. Details of the impact

The **commercial** impact of our work has three interrelated components: 1) the **development and commercial adoption of PENCOR technology** by Allianz SE; which led to 2) the **mitigation of potential future financial losses** for Allianz and its customers through technology-enhanced risk assessment; and 3) significant **gains in productivity and cost-effectiveness** through Allianz's ability to mitigate opportunity losses for its global customer base.

1) <u>Developing PENCOR, a new technology commercially adopted by Allianz SE</u>

Kočvara's new optimisation technology embodied in PENCOR has directly resulted in the **research-led enhancement of operations practice of the world's 5th largest insurance business (S4), Allianz SE**, which operates in 79 countries (S5). Kočvara materially contributed to this success through Allianz's adoption of PENCOR in 2014.

Allianz's Group Risk Division approached Prof. Kočvara to work on the nearest correlation matrix problem for their Internal Risk Capital Modelling. Kočvara's application of **a constraint on the matrix condition number** (the **key finding** of the underpinning research) led to the creation of PENCOR, an optimiser which is able to precisely quantify economic risk. The software offers a **much more stable solution** than the standard formulation (S1 and S3), providing a significant commercial advantage to Allianz. Allianz adopted the technology in 2014 as part of its strategy to ensure compliance with the EU's Solvency II regulatory framework.

The regulatory framework came into effect on 1 January 2016. However, Allianz adopted the technology in advance, as it began to prepare for the introduction of the regulation once it was adopted by the European Parliament and Council of the European Union in 2009 (S6). Solvency II requires the quantification of all risks by every insurance company headquartered in the EU (S2). This entails comparing the quantified risk number to available risk capital (a portion of total equity), i.e. the liquid capital that needs to be available to cover investment losses. Kočvara developed PENCOR to quantify this risk and to contribute to Allianz meeting this requirement efficiently.

Allianz now uses PENCOR in an early stage of the construction of economic scenarios to evaluate this market risk, which is associated with nearly €23 billion of risk capital (S2, p. 83). In these scenarios, the correlation of thousands of risk factors is modelled and collated in a nearest correlation matrix, which is found via the PENCOR optimiser. This matrix is signed off by a corresponding parameter committee (S2) who take responsibility for the parameters inputted to the internal risk model.

The former Senior Risk Analyst at Allianz confirmed the importance of PENCOR to these internal preparations for regulatory compliance. It 'is absolutely mandatory to have a stable and reliable process in place to determine the underlying correlation matrix. This is one of PENCOR's main strengths: it is able to provide realistic, albeit stable correlation matrices, even over the course of several years, which in turn lead to rather stable risk capital numbers' (S3). PENCOR has also sped up internal processes at Allianz, leading to considerable time savings: 'the computation time of [the] PENCOR approach is significantly shorter than that of any benchmark processes' (S1) and it 'saves roughly 20 person days a year' (S3).

2) <u>Enabling sustainability of Allianz into the future by improved methods of risk assessment that ensure regulatory compliance</u>

PENCOR directly contributes to regulatory compliance through advanced risk assessment. As noted above, Kočvara's key finding of a constrained formulation of the NCM problem means that the PENCOR optimiser 'is superior to benchmark approaches' (S1), producing a much more stable solution than the standard formulation (S3). This stability of the NCM results in stable risk capital numbers over longer time-frames. This is important because it means that Allianz needs fewer liquid assets to meet regulatory requirements. PENCOR ensures the stability of risk capital parameters, which are key to **driving down the amount of risk capital that Allianz is required to hold as a buffer**. The required amount of risk capital in the Solvency II framework is called the Solvency Capital Requirement (SCR).

If the insurer's available resources fall below its SCR, then supervisors are required to take action to restore the insurer's finances to the level of the SCR. If this is not done, the level of supervisory intervention is progressively intensified. Ultimately, if the available resources of the insurer fall below the Minimum Capital Requirement (MCR), then the insurer's liabilities are transferred to another insurer, and the license of the insurer is withdrawn, or the business is liquidated (S7).

Compliance with Solvency II was considered difficult, with the implementation date postponed several times (S6). By forming a critical part of the internal risk calculation for Pillar 1 of Solvency II, **PENCOR has helped Allianz meet this challenging regulatory requirement**. Such regulatory compliance ensures that Allianz's global trading can continue unimpeded.

3) Delivering significant gains in productivity and improved cost-effectiveness, saving Allianz an estimated €57 million in opportunity losses since 2016

By reformulating the nearest correlation matrix problem as an NLP-SDP with a matrix condition number constraint, optimisation with the new technology PENCOR has improved Allianz's risk number calculation by making it much more stable. This has resulted in significant productivity gains. The nearest correlation matrix is one risk capital parameter among many: others include volatilities, exposure, mitigating measures. According to a former Senior Risk Analyst from the Risk Methodology Group at Allianz (the team that laid the methodological foundation for their current internal risk capital model), a stable calculation of the nearest correlation matrix decreases the value of the risk capital typically by around 0.5% (range from 0.1%–5%, depending on market conditions; S3).

The risk capital for market risk of Allianz is currently around €23 billion (S2, p. 83). As such, the 0.5% improvement in performance that PENCOR drives by stable construction of the nearest correlation matrix equates to a decrease in the risk capital of around **€115 million each year since the beginning of 2016**. PENCOR has therefore freed up an estimated total of 5 x €115 million = **€575 million for investment** this impact period.

Since risk capital cannot provide any return on investment, the use of PENCOR has therefore **mitigated opportunity losses** for Allianz, and the global customers for which it manages assets, of around **€11.5 million each year since 2016** (assuming a realistic yield differential of 10% between risk-free rate and return on equity; S3).

Thus, PENCOR has provided **significant gains in productivity** to Allianz by mitigating an estimated total of 5 x €11.5 million ≈ **€57 million** opportunity losses for Allianz and their global customer base in the current impact period.

5. Sources to corroborate the impact

S1. Letter from the Group Risk Division at Allianz SE (dated 12 Dec. 2019).

S2. Annual Report 2018, Allianz group (English Language version).

S3. Letter from the former Senior Risk Analyst in the Risk Methodology Group at Allianz (dated 6 July 2020).

S4. Insurance Business, 'These are the top 25 largest insurance companies in the world' [accessed 18 Dec. 2020].

S5. Allianz, 'Find us in your own location' [accessed 18 Dec. 2020].

S6. RIMES White Paper "Solvency II The Data Challenge" [accessed 18 Dec. 2020].

S7. Solvency II FAQs [accessed 18 Dec. 2020].