Institution: Brunel University London		
Unit of Assessment: 10 Mathematical Sciences		
Title of case study: Automatic Bayesian quantile regression via SAS		
Period when the underpinning research was undertaken: 2005-2013		
Details of staff conducting the underpinning research from the submitting unit:		
Name(s): Keming Yu	**Role(s) (e.g. job title):** Professor	**Period(s) employed by submitting HEI:** 09/2005-present
Period when the claimed impact occurred: August 2013 - December 2020		
Is this case study continued from a case study submitted in 2014? N		

1. Summary of the impact (indicative maximum 100 words)

Following the 2007 financial crisis, modelling of financial risk became a significant issue in banking and finance. Quantile-based risk measures are more sensitive to events that happen in the tail end of a distribution – the tail risk. Bayesian methods allow prior knowledge of the process to be incorporated into the analysis. Bayesian quantile regression (BQR) was first introduced by Prof Yu and has been available for risk analysis by the millions of users of the statistical software suite SAS since 2009. The SAS software is currently installed at 83,000 business, government and university sites worldwide. BQR has been particularly useful to banks and made available to them through the R and SAS software packages. For instance, the People's Bank of China were able to increase their investments in individual businesses year-on-year because with the BQR model they were more confident about the profit on each investment. In one year alone (2015), they used Prof Yu's model to make decisions on 20,000 business cases worth GBP110,000,000. The China Minsheng Bank believe the use of more reliable risk management systems enabled by BQR has allowed them to provide loans to 2,000 more customers per annum.

2. Underpinning research (indicative maximum 500 words)

Many activities in science, social science, engineering and finance involve comparison such as one method compared to another, new techniques compared to traditional ones, the modelling of relationships between or among factors, and the prediction or forecasting of a future state. Quantiles are points in a distribution that relate to the rank order of values in that distribution. A quantile-based risk measure is more sensitive to events that happen in the tail of a distribution – the tail risk. For example, Value at Risk has become the standard measure of market risk employed by financial institutions for both internal and regulatory purposes. Despite its conceptual simplicity, its measurement is a very challenging statistical problem and none of the methodologies developed so far give satisfactory solutions. Interpreting Value at Risk as a quantile of future portfolio values conditional on current information moves the focus of attention from the distribution of returns directly to the behaviour of the quantile. Bayesian methods, nowadays widely used in astronomy, economics, marketing, genetics, bioinformatics and social sciences, allow prior knowledge of the relevant data characteristics to be incorporated into the statistical analysis. Prof Yu with his co-authors first explored quantile regression with Bayesian inference. (Ref 1, Ref 2). The model and method became available in SAS in 2007-2008 (E5) and available in R (https://cran.r-project.org/web/packages/Brq/Brq.pdf) were discussed in the

research paper (Ref 2) and research papers (Ref 3, Ref 4) respectively. Previously, in 2006, Prof Yu organised the "Quantile Regression, LMS Method and Robust Statistics in the 21st Century" EPSRC and ICMS jointly funded workshop, held in Edinburgh, in which participants from leading universities and industries around the world were provided with an overview of the state of research in quantile regression.

In 2013, under the support of the NIHR Research Methods Opportunity Funding Scheme (NIHR-RMOFS-2013-03-09) for the project `Exploring the role of dietary and physical activity behaviours along the BMI distribution: a Bayesian quantile regression approach', Prof Yu then collaborated with health economics researchers to apply quantile regression method for health data analysis.

3. References to the research (indicative maximum of six references)

Ref 1: Yu, K. and Stander, J. (2007) Bayesian analysis of a Tobit quantile regression model, *Journal of Econometrics,* 137(1): 260-276 10.1016/j.jeconom.2005.10.002

Ref 2: Chen, C. and Yu, K (2009) Automatic Bayesian quantile regression curve fitting, *Statistics and Computing,* 19(3): 271-281 10.1007/s11222-008-9091-x

Ref 3: Al-Hamzawi, R., Yu, K. and Benoit, D.F. (2012) Bayesian adaptive LASSO quantile regression, *Statistical Modelling*, 12(3): 279-297. 10.1177/1471082X1101200304

Ref 4: Yu K, Chen C.W.S, Reed C. and Dunson, D.B. (2013) Bayesian variable selection in quantile regression, Statistics and Its Interface, 261-274. 10.4310/sii.2013.v6.n2.a9

4. Details of the impact (indicative maximum 750 words)

Since the financial crisis, there have been major changes in the regulation of large banks directed at reducing their risk. Measures of regulatory capital have substantially increased; leverage ratios have been reduced; and stress-testing has sought to further assure safety by raising levels of capital and reducing risk-taking. In recent years, Value at Risk has become a popular tool in the measurement and management of financial risk. However, there is accumulating evidence that financial time series and return distributions are not well approximated by Gaussian models or normal distributions. In particular, it is frequently found that market returns display negative skewness and excess kurtosis. Extreme realizations of returns can adversely affect the performance of estimation and inference designed for Gaussian conditions; this is particularly true of ARCH and GARCH models whose estimation of variances is very sensitive to large innovations. For this reason, financial risk measurement has recently shifted attention toward the development of more robust estimators of conditional quantiles. Prof Yu's models and methods are particularly welcomed by the banking sector, and incorporated into SAS for the benefit of their millions of users, because they combined quantile regression and Bayesian inference-based decision making for risk management. Bank's profits are generated by offering services to customers and by taking investment risks with the general assumption that the greater the risk, the greater the profit. However, empirical data sometimes contradicts this norm and the relationship between risk and return has proven to be a long-standing research problem.

The People's Bank of China, which is the central bank of the People's Republic of China responsible for carrying out monetary policy and regulation of financial institutions in mainland

China, has identified 2 significant advantages of using the SAS Bayesian Quantile Regression package compared to the methods they used previously. Prof Yu's model has allowed them to determine the consistency of the risk-return relationship of their business across all levels of profitability, departing from the more conventional approach of looking at the relationship dynamics between risk and bank performance. This means that every corporate decision that can lead to profits is based on Prof Yu's model of risk assessment due to its accuracy. In 2015, the People's Bank of China [E1] used Prof Yu's algorithm to make corporate decisions on 20,000 business cases worth GBP110,000,000. They saw their profits increase by about 12% per annum from 2014 to 2018. This was based on an increase in profits from CNY8,000,000, equivalent to GBP904,431 (01-2021), to CNY18,000,000, equivalent to GBP2,034,163 (01-2021) between 2010 and 2018. Additionally, the first private bank in China, Minsheng Bank [E2], owned mostly by non-government enterprises, demonstrated Professor Yu's advanced risk measurement models to its customers recently and as a result of this saw their customer numbers increase by 2,000 per annum from 2016.

Other Impacts

The Chinese governmental authorities hired the Wang Yanan Institute for Studies in Economics to develop user-friendly software in R and SAS for banks and financial institutions. The Wand Yang Institute is one of the top centres for teaching and research on Economics and Finance in China. Professor Yu's quantile regression method was included and particularly illustrated. [E3 and E4]. SAS recognised that Prof Yu as a leading researcher in Bayesian quantile regression (BQR) methods and SAS software 'has implemented the Bayesian inference and quantile regression methods that were proposed by Dr. Keming Yu and his collaborators…The Bayesian inference methods have been attracting increasing interests within the pharmaceutical industry at various stages of the research, development, manufacturing, and health economic evaluation of new health care interventions.' [E4]

In particular, they recognised its high applicability in the medical sector: 'typical clinical trials for progressive diseases often observe excessive declines in efficacy endpoints due to rapid disease progression including deaths or loss of capability to perform the efficacy assessment.' Professor Yu's quantile regression provides 'robust and power tools for dealing with the issues in clinical trials' and they have published an example guidance on how to perform Bayesian quantile regression by using SAS/STAT software's MCMC procedure. [E5] This has enabled banks and financial institutions in China to access and use Prof Yu's algorithm to estimate risk.

Prof Yu's quantile regression method has been coded into the GAUSS Mathematical and Statistical System [E6]. The GAUSS Mathematical and Statistical System is a data analysis, mathematical and statistical environment based on the powerful, fast and efficient GAUSS Matrix Programming Language. It has been widely used by scientists, engineers, statisticians, biometricians, econometricians, and financial analysts for more than 30 years.

5. Sources to corroborate the impact (indicative maximum of 10 references)

E1) Corroborating letter from People's Bank of China.
E2) Corroborating letter from China Minsheng Bank.
E3) Cai, Z. (2007) "Advanced Topics in Analysis of Economic and Financial Data Using R and SAS 1." (PDF provided)
E4) Corroborating letter from SAS

E5) SAS/STAT software's guidance example on MCMC procedure https://support.sas.com/rnd/app/stat/examples/BayesQuantile/quantile.htm PDF provided.
E6) Qreg: A GAUSS Library for Computing Quantile Regression: http://www.thierry-roncalli.com/download/gauss-qreg.pdf PDF provided.