Institution: Aberystwyth University

Unit of Assessment: 10: Mathematical Sciences

Title of case study: Inspiring the next generation of Mathematicians in Wales using the optimal geometry of soap films

Period when the underpinning research was undertaken: 2010- 2020

Details of staff conducting the underpinning research from the submitting unit:

Name(s):	Role(s) (e.g. job title):	Period(s) employed by submitting HEI:
Professor Simon Cox	Reader; Professor	1 December 2006- present
Dr Tudur Davies	Post-Doctoral Research Associate; Lecturer	1 May 2010- present

Period when the claimed impact occurred: 2015 - present

Is this case study continued from a case study submitted in 2014? N

1. Summary of the impact (indicative maximum 100 words)

Aberystwyth University (AU) research on the optimal arrangements of soap bubbles and soap films is utilised as a vehicle for public engagement in mathematics. Presentations and demonstrations have been given in both Welsh and English at various events, such as school visits and national festivals. These have had an impact on the awareness and interest of school children in geometry and mathematics, and have also inspired teachers to consider different ways of teaching mathematics. The activities have been used as part of AU's efforts to widen access and to enhance participation with research in the mathematical sciences.

2. Underpinning research (indicative maximum 500 words)

A soap film naturally minimises its surface energy, which is directly proportional to its surface area, due to surface tension. As a consequence of this energy minimisation, collections of soap films and bubbles, that form an aqueous foam, have a structure at equilibrium that is precisely defined and that satisfies Plateau's laws and the Young-Laplace law, which relate to their topology and geometry. Plateau derived his laws for optimal geometries of soap films and foams in the 19th century by using wire frames dipped in soap solution. The usefulness of this technique for demonstrating fundamental concepts in minimisation and geometry continues to this day, acting as a spur to both scientific endeavour and public engagement. The development of mathematical models of the structure and dynamics of aqueous foams, is of particular relevance to industries that include ore separation, soil remediation and oil recovery.

AU outreach activities demonstrate that the energy minimisation driving a foam to an equilibrium state provides a visualisation of the solution of problems in variational calculus. For example, soap films can be used to solve a Steiner problem to find the minimum length way of connecting n vertices, which in the real world could denote towns, cities or network hubs. This problem in length minimisation gives rise to the following question on the optimal geometries of soap bubbles in the plane: for N bubbles of given areas, which arrangement has least perimeter, and hence energy? This is a packing problem, the solution to which can indicate how best (in the sense of least interface, and possibly least deformation) to fit deformable objects within some given boundary. Ongoing research in AU seeks to provide candidate solutions to this problem [3.1; 3.2]. We also consider the least area way of partitioning geometric shapes such as a sphere or a cylinder [3.3].

The geometrical rules that an optimal geometry for a cluster of bubbles satisfies can also be used as the basis of predictions of the rheology of foams. The AU research group uses energy minimisation to simulate the structure, the static rheology, and the slow (quasi-static) flow of foams [3.4; 3.5], for example in their EPSRC funded work [3.6].

3. References to the research (indicative maximum of six references)

3.1 S.J. Cox and E. Flikkema (2010) The minimal perimeter for N confined deformable bubbles of equal area. Elec. J. Combinatorics 17:R45 DOI: 10.37236/317

3.2 F.J. Headley and **S.J. Cox** (2019) Least-perimeter partition of the disc into N regions of two different areas. Euro. Phys. J. E. 42: 92. DOI: 10.1140/epje/i2019-11857-0

3.3 I.T. Davies, L. Garratt and **S.J. Cox** (2015). Rhaniad arwynebedd lleiaf silindr yn dair rhan. Gwerddon, 20, October 2015, 30-43. URL: www.gwerddon.cymru/en/editions/issue20/article2/

3.4 D. Vitasari, **S. Cox**, P. Grassia and R. Rosario (2020). Effect of surfactant re-distribution on the flow and stability of foam films. Proc. R. Soc. A. 476: 20190637. DOI: 10.1098/rspa.2019.0637

3.5 I.T. Davies (2018). Simulating the interaction between a descending super-quadric solid object and a soap film. Proc. R. Soc. A., 474(2218): 20180533. DOI: 10.1098/rspa.2018.0533

Research grants

3.6 EPSRC Grant, EP/N002326/1 Flow of Gas-Liquid Foams in Narrow Complex Geometries, 2016 – 2019, GBP254,962

4. Details of the impact (indicative maximum 750 words)

AU researchers Cox and Davies have developed a number of research-related talks and demonstrations on the geometry of soap bubbles to explain the mathematics of minimisation to secondary school students. Since 2015, they have further developed their existing engagement activities for schools and festivals with the following objectives in mind:

(i) inspiring learners to consider further study in mathematics as an attractive option;
(ii) supporting teachers by sharing ideas about visual ways in which mathematics can be demonstrated;
(iii) providing online material to enhance participation across Wales.

The activities involve:

a) demonstrating Plateau's laws with wire frames, including hysteretic transitions between different local minima;
b) least area arrangements of soap films, comparing the solutions [3.1] with intuition;
c) a discussion of soap-film solutions of Steiner-like problems in the plane (e.g. the shortest road network joining different towns/cities in Wales) and local minima in complicated energy landscapes. Numerical solutions were used to indicate the relevance of constraints (e.g. mountains).

Recent activities have targeted a Welsh medium audience in an effort to promote the benefits of studying Mathematics to a higher level through the medium of Welsh. Providing activities in Welsh is important, as it shows to teachers and students that there are no barriers to discussing mathematics through the medium of Welsh. Some of the schools that are visited are in a rural location (for example, Ysgol David Hughes on Anglesey and Ysgol Brynhyfryd in Denbighshire; see also Figure 1 below), and therefore, the students do not have easy access to scientific events of this type. Such activities are therefore vital to ensure that aspirations of school children are not limited by their location.

One career pathway that we promote in our activity is teaching. The Welsh Government have a target of more than doubling the number of secondary school teachers who are able to teach through the medium of Welsh by 2050, as they aim to increase the number of Welsh speakers to a million. Their incentive schemes [www.discoverteaching.wales/teacher-training-incentives/] clearly indicate that increasing the number of high-quality teachers of mathematics is a priority (with training grants between GBP20,000 and GBP6,000 for high priority subjects), and in particular those who can teach through the medium of Welsh (additional training grant of up to GBP5,000).

The AU developed outreach activity has been presented to more than 200 secondary school learners during 10 school visits across Wales and the Marches, for which teachers have returned very positive feedback, commenting that learners were inspired by the session and that they noticed a positive impact on their interest in mathematics [5.1.1- 6]. The availability of the activity through the medium of Welsh was particularly appreciated, with one teacher commenting that the activity *'showed to learners that it is possible to discuss high level mathematical concepts in Welsh'* [5.1.5]. The activity has also inspired some of the teachers who participated, with one commenting that:

> *"It has inspired me to think about how I can present other mathematical topics in a similar way. It offered me an example of how to present mathematical problems in a fun and interesting context. I am more aware of how maths and research apply outside the classroom, and now make regular links between them in lessons to try to enrich the learners' experience."* [5.1.6].

The activity has also been presented at many Further Mathematics Support Programme Wales (FMSPW) conferences, thus reaching a wider audience, and contributing to the FMSPW being able to offer enrichment sessions through the medium of Welsh [5.2.1]. The sessions *'instigated some interesting discussions and it was clear that learners were inspired by the activity'* from which *'teachers also benefited'* [5.2.1]. The activity was adapted as 50-minute videos, one in Welsh and the other in English, to be part of the FMSPW's "Bridging A-level to University" programme, which contributed to a large number of Welsh Year 13 students reconnecting with learning mathematics during Covid-19 restrictions. The FMSPW provided positive feedback on the bridging session, saying that it

> *"was a significant part of our successful attempt to bridge between school and university given that many of these Y13 students would not have the opportunity to widen their mathematics experiences because of the radical effect of the pandemic on their schooling"* [5.2.1].

Approximately 200 students from over 30 schools participated in the programme overall, with over 1,000 views of the videos in the series, which helped the students to continue learning about Mathematics during lockdown [5.2.2].

The AU research team have also taken their activity to seven national Eisteddfodau (four main ones and a further three Urdd festivals), where at least 280 participants have interacted with our activity at sessions hosted by AU and the Coleg Cymraeg Cenedlaethol (CCC) [5.3; 5.4]. The National Eisteddfod is the main cultural festival in Wales (attendance approximately 160,000), while the Urdd Eisteddfod is the largest youth festival (attendance approximately 90,000) and they are amongst the largest of their type in Europe. The activity has proven popular at these Eisteddfodau, as noted in the testimonial letter from the Urdd which states that *'the attraction of bubbles always manages to stimulate dozens of children and their families to have a go at solving mathematical problems while they wander the festival site'* [5.4]. The activity is part of a wider collaboration between the Urdd and Aberystwyth University, which has led to a growth in STEM activities offered at the festival with the result that the *'profile of mathematics amongst our audience has certainly increased by having a presence in the GwyddonLe and other stands on the field'* [5.4]. The CCC view the activity as a valuable tool to promote opportunities to study

mathematics through the medium of Welsh *'in a visual way and to people of all ages and backgrounds'* [5.3]. The activity was also delivered at the Soapbox Science public event in Swansea in June 2018 where its visual nature again proved to be popular, with one child commenting that the activity *'makes me want to be a scientist'* [5.5].

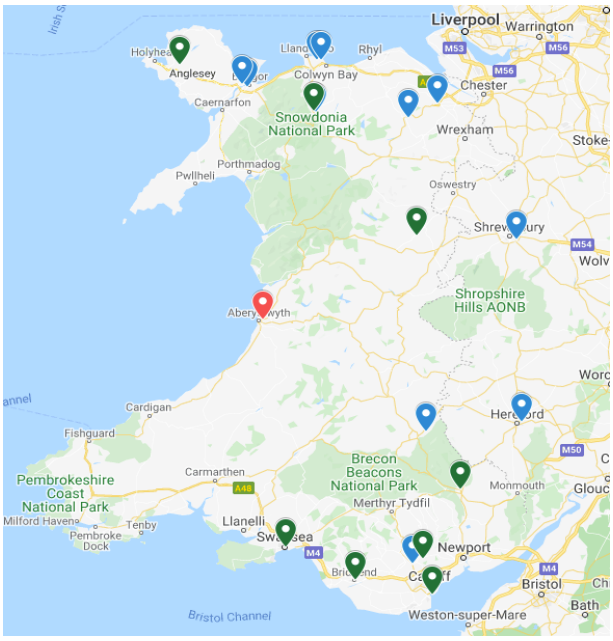

Figure 1: Locations of school visits (blue marker), internal school conferences (red marker) and other public events such as the National Eisteddfodau (green marker).

As a result of Covid-19 restrictions, the activity has been adapted into an online learning resource [5.6.1], which is available through resource portals set up by Aberystwyth University and the CCC as well as indirectly through the Welsh Government's own resources Hub. The resource attracts 10-20 new users each month. [5.6.2]

One of the research papers listed above [3.3] was the first mathematical paper published in the Welsh medium journal *Gwerddon* [5.7]. This paper is for a more general audience than is usual for scientific journals, and therefore brings knowledge of foam structure to a new audience. It is currently the most-viewed Mathematics article on the CCC Resource Portal [www.porth.ac.uk]. [5.8] This paper again demonstrates that there is no reason why high-level mathematics cannot be discussed through the medium of Welsh.

5. Sources to corroborate the impact (indicative maximum of 10 references)

5.1 School testimonials

5.1.1 Testimonial from Head of Mathematics, Ysgol y Creuddyn (Conwy County) [Lang.: Welsh]
5.1.2 Testimonial from Mathematics Teacher, Ysgol Brynhyfryd (Denbighshire) [Lang.: Welsh]
5.1.3 Testimonial from Head of Mathematics, Ysgol Maes Garmon (Flintshire) [Lang.: Welsh]
5.1.4 Testimonial from Head of Mathematics, Ysgol Tryfan (Gwynedd) [Lang.: Welsh]
5.1.5 Testimonial from Mathematics Teacher, Coleg Meirion Dwyfor (Gwynedd) [Lang.: Welsh]
5.1.6 Testimonial from Mathematics Teacher, Ysgol David Hughes (Ynys Môn) [Lang.: Welsh]

5.2 Further Mathematics Support Programme in Wales testimonials

5.2.1 Testimonial from Area Coordinator Mid Wales, Further Mathematics Support Programme in Wales
5.2.2 Testimonial from Programme Leader, Further Mathematics Support Programme in Wales

5.3 Testimonial from Chief Executive, Coleg Cymraeg Cenedlaethol [Lang.: Welsh]

5.4 Testimonial letter from Assistant Organiser of the Eisteddfod and Arts Department, Urdd Gobaith Cymru [Lang.: Welsh]

5.5 Twitter @*OAHowells*: https://twitter.com/OMHowells/status/1010495147388428288

5.6 "Mathematics with Bubbles" website
5.6.1 "Mathematics with Bubbles" website: users.aber.ac.uk/itd/MATHS-BUBBLES/
5.6.2 Google analytics for "Mathematics with Bubbles" website

5.7 "Gwerddon yn torri tir newydd". www.gwerddon.cymru/cy/ffrwd/newyddion/name-9133-cy.aspx [Lang.: Welsh]

5.8 Views of Mathematics article on the CCC Resource Portal