

Institution: Manchester Metropolitan University		
Unit of Assessment: C23 Education		
Title of case study: Making Sense of Mathematics: Increasing inclusion and achievement in schools and colleges through Realistic Mathematics Education		
Period when the underpinning research was undertaken: 2004-2020		
Details of staff conducting the underpinning research from the submitting unit:		
Name(s):	Role(s) (e.g. job title):	Period(s) employed by submitting HEI:
Yvette Solomon	Professor	2009-present
Frank Eade	Principal Lecturer	1983-2011
Susan Hough	Senior Lecturer	2008-present
Stephen Gough	Senior Lecturer	2008-present
Paul Dickinson	Senior Lecturer	1996-present
Una Hanley	Senior Lecturer	1990-2012
Harry Torrance	Professor	2003-present
Vinay Kathotia	Senior Research Associate	2018-present
Period when the claimed impact occurred: 31 August 2013-31 December 2020		
Is this case study continued from a case study submitted in 2014? No		
<p>1. Summary of the impact</p> <p>A mathematics curriculum materials design programme has been developed over 15 years at Manchester Metropolitan University in an iterative process of research, development, evaluation and impact. Focusing on inclusion of all students in mathematics, this programme has enhanced practitioners' understanding of, and enthusiasm for, mathematics teaching, and raised student engagement and achievement as a result of involvement in large-scale projects involving 120+ schools/colleges and 17,000+ students since 2013. It has trialled materials and developed teaching methods. Associated teacher training in Maths Hubs and other professional development contexts is extensive, involving direct engagement with approximately 1,000 teachers, and indirect impact on many more, through professional development design. Drawing on over GBP2,500,000 of research funding, the programme has changed local and national education policy/practice in mathematics education and teacher education in England, the Cayman Islands, Norway and India.</p>		
<p>2. Underpinning research</p> <p>The underpinning research began in response to national and international reports and surveys (PISA, TIMSS). These raised concerns about mathematical achievement in England and indicated the potential of the Realistic Mathematics Education (RME) approach in the Netherlands to address poor conceptual understanding and problem-solving skills. Based on Freudenthal's theory of emergent mathematics, an RME curriculum had been developed by Heuvel-Panhuizen and colleagues at Utrecht and adapted for the US context by the University of Wisconsin (Maths in Context). Collaborating with Wisconsin, the Manchester Metropolitan team researched how students engaged with mathematics when using the US materials in a three-year longitudinal study of 22 local Key Stage 3 classrooms plus a national trial in eight schools (Maths in Context (MiC), GBP515,000, The Gatsby Foundation, led by Eade and Dickinson, 2004-2007). Analysis of approximately 1,700 students' work suggested that RME encouraged development of informal strategies that enhanced understanding and facilitated problem-solving [1].</p> <p>The research programme was taken forward into Key Stage 4, in Making Sense of Maths (MSM) (GBP46,975, Mathematics in Education and Industry, and Esmée Fairbairn Trust, 2007-2009). Led by Eade and Dickinson, but also involving Hough and Gough, both previously school-based teacher-researchers in the MiC project, MSM developed and trialled materials with the support of the Freudenthal Institute in Utrecht. Subsequently published by Hodder as a series of textbooks, teacher guides and workbooks (25,000 copies sold since 2014), the MSM materials were trialled with Key Stage 4 Foundation-level pupils in 12 schools involving 28 teachers. Teachers reported that students were more engaged, had improved understanding and were more able to apply mathematical models [1].</p> <p>The trial of the RME approach and MiC materials provided a significant opportunity to study curriculum 'borrowing' in action through a parallel ESRC-funded project called Investigating Effective Strategies for Maths Teaching at Key Stage 3 (GBP46,536, ESRC RES-000-22-</p>		

1082, led by Hanley and Torrance, 2005-6). This focused on the MiC project mathematics teachers' take-up of RME materials and pedagogies. Alert to the fact that pedagogic materials are not easily imported from one national domain to another, the researchers investigated how teachers translated RME to make it work in the context of the English curriculum, finding that they customised the training input, working in locally-situated ways to embed RME into their existing practice [3, 4, 5].

The success of MSM with Foundation-level students indicated the potential of RME at post-16 GCSE resit level, where traditional approaches frequently lead to repeated failure and major implications for employment, training and social inclusion (cf. the Smith Report, 2017). Led by Hough and Solomon, **Investigating the impact of a Realistic Mathematics Education approach in Post-16 GCSE resit classes** (GBP82,202, Nuffield Foundation, 2014-2016) trialled RME in four post-16 GCSE resit classes, using a quasi-experimental design to demonstrate enhanced sense-making and student engagement [2, 6].

Recognising the potential of RME for targeting socio-economically disadvantaged students by reducing under-performance in mathematics, a further iteration of materials and trialling was funded by the Education Endowment Foundation (EEF) in **Realistic Maths Education at KS3** (GBP844,093, 2018-2021). This is a randomized controlled trial involving 119 schools, led by Solomon. This project trained 120+ Key Stage 3 teachers in 60 schools to use RME materials, testing student performance in a standardised test in Year 9 (delayed from Year 8 due to COVID-19) in comparison to 'business as usual' teaching in 59 control schools. Additional qualitative measures explored the impact of the training on teachers' practices and beliefs, and their understanding of the nature of mathematics learning.

Recognition of the inclusive potential of RME led to three further international projects. GCRF funded the **Grounded and expressive mathematics education collaboration** (GBP11,950, 2019, led by Kathotia), exploring the potential for development of RME in deprived socio-economic contexts in Rajasthan, India, followed by **Fostering Research Capacity and Inclusive Mathematics Education in India** (GBP20,810, 2019-20, led by Kathotia). The Norwegian Research Council funded **Inclusive Mathematics Teaching: Understanding and developing school and classroom strategies for raising attainment (IMaT)**, which includes a teacher development component based on RME, drawing on Manchester Metropolitan expertise and materials (NOK12,000,000 [GBP1,091,000], 2019-2022, led by Solomon).

3. References to the research

1. Dickinson, P., Eade, F., Gough, S., Hough, S. & Solomon, Y. (2019) Implementing RME in England and the Cayman Islands – dealing with clashing educational ideologies, in Marja Van den Heuvel-Panhuizen (ed) [International reflections on the Netherlands Didactics of Mathematics](#), Springer.
2. Hough, S., Solomon, Y., Dickinson, P. and Gough, S. (2017) [Investigating the impact of a Realistic Mathematics education approach on achievement and attitudes in Post-16 GCSE resit classes. Final Report to the Nuffield Foundation](#), Manchester: Manchester Metropolitan University.
3. Hanley, U. and Torrance, H. (2011) 'Curriculum Innovation: difference and resemblance' [Mathematics Teacher Education and Development](#) 13, 2, 67-84.
4. Hanley, U., Darby, S., and Torrance, H. (2007). [Final report – Investigating and developing effective strategies for mathematics teaching at Key Stage 3 in the English National Curriculum](#) (ESRC Ref: RES-000-22-1082), Manchester Metropolitan University.
5. Hanley, U. and Darby S. (2006) Working with curriculum innovation: teacher identity and the development of viable practice. [Research in Mathematics Education](#), 8, 53-66. DOI:10.1080/14794800008520158
6. Solomon, Y., Hough, S. & Gough, S. (2020) The role of appropriation in guided reinvention: establishing and preserving devolved authority with low-attaining students, [Educational Studies in Mathematics](#) DOI:10.1007/s10649-020-09998-5

Key research grants:

- G1 Maths in Context (MiC), Eade and Dickinson, The Gatsby Foundation, 2004-2007, GBP15,000.
- G2 Investigating Effective Strategies for Maths Teaching at Key Stage 3, Hanley and Torrance, ESRC (RES-000-22-1082), 2005-6, GBP46,536.

- G3 Making Sense of Maths (MSM), Eade, Dickenson, Hough and Gough, Mathematics in Education and Industry and Esmée Fairbairn Trust, 2007-2009, GBP46,975.
- G4 Investigating the impact of a Realistic Mathematics Education approach in Post-16 GCSE resit classes, Hough and Solomon, Nuffield Foundation, 2014-2016, GBP82,202.
- G5 Realistic Maths Education at KS3, Solomon, Education Endowment Foundation (EEF), 2018-2020, GBP736,322 plus GBP107,771 COVID-19 extension to 2021.
- G6 Grounded and expressive mathematics education collaboration, Kathotia and Solomon, GCRF, 2019, GBP11,950 and Fostering Research Capacity and Inclusive Mathematics Education in India, GCRF 2019-20, GBP20,810.
- G7 Inclusive Mathematics Teaching: understanding and developing school and classroom strategies for raising attainment (IMaT), Solomon, Norwegian Research Council, 2019 – 2022, NOK11,994,000 (GBP1,091,000).

4. Details of the impact

Manchester Metropolitan's RME materials development and the accompanying CPD model have had a substantial impact on practice in schools, colleges and in-service teacher education in England and the Cayman Islands. It has also influenced practice in India, where GCRF-funded research visits in 2019 and 2020 to discuss and develop the use of an RME approach in schools, targeting disadvantaged students, resulted in staff development in the understanding of support for mathematical literacy.

Since August 2013, thousands of students in England have been the immediate beneficiaries of the research due to their involvement in the Nuffield-funded GCSE resit trial (147 students) and the EEF-funded randomised controlled trial (approx. [17,000 KS3 students](#)). Independent analysis of pre-post test data in the GCSE resit project showed that the use of RME models correlated with improved performance among the intervention group in use and understanding of number and proportional reasoning [2, Appendix 8]. Detailed script and classroom analysis showed enhanced sense-making [2], and the potential of RME for re-engaging low attainers [6]. In the EEF trial, teachers report on the benefits of the materials for deeper learning (*"I think it really helps to embed the concepts and ensures pupils understand the work ... I'm sure if pupils were taught this way from early years we wouldn't struggle with teaching and embedding the fundamentals as much in secondary school. Instead of trying to reverse poor learning we would be able to help pupils make progress with deeper learning."* [Manchester Teacher]) and on teachers' practice (*"RME questioning has had an impact on improving questioning in many teachers' lessons across all teaching groups."* [Mansfield Teacher]). To date, at least 20 out of 60 schools in the EEF trial have taken up RME beyond trial requirements [A], integrating RME into their schemes of work at KS3 and beyond, cascading RME across whole year groups, or rolling it out to GCSE level, as described by the Head of Mathematics at Hadlow Rural School, Kent: *"We have continued to use the Year 7 material with our new 2020 cohort (not a requirement of the programme) as we wholeheartedly believe that this approach has a positive impact on both the learning and enjoyment of our students.... Schemes of work across both Key Stage 3 and 4 continue to be developed to draw on the lessons learned from RME and the pedagogy behind it."* [B]

The advent of COVID-19 in spring 2020 led to an extension of the trial activity and redirection towards (i) support for remote teaching of students at home and (ii) production of optional 'recovery' materials in a capstone unit for Year 9, targeting missed schooling due to lockdown. Teachers continued to attend optional training by Zoom through to December 2020 (N=80) [C]. The final release materials will become open access from June 2021 on a dedicated legacy [website](#).

The success of Manchester Metropolitan's work has led to widespread take-up in England of the RME approach and resources. *Making Sense of Maths* (Hodder) has sold 25,500 copies since January 2014 [D]. As one of the schools in the original MiC project, Cheadle Hulme High School introduced a department-wide RME approach, and still uses materials written by the Manchester Metropolitan team, including new materials produced in the EEF trial. The school is the [highest performing in mathematics in Greater Manchester](#) and its [curriculum description](#) refers to RME hallmark pedagogy, e.g. use of context and drawing, multiple strategies and progressive refinement. The school is now part of the Laurus Trust Academy Trust,

implementing RME across its five secondary schools, comprising approximately 4,000 pupils. Director of Mathematics Marisa Bartoli writes: *“GCSE results ... went from strength to strength and whilst many factors contributed to this, mock results showed that students were attempting questions that traditionally might have been left unanswered. One of the schools that has joined the Trust is a fully established school and colleagues have been directed to focus on modifying their more traditional approach ... The structure that the EEF project resources provide frees them to focus on the questioning...parents have indicated that their students have already begun to reflect positively on the change to their Maths lessons this year, suggesting that they are “happier in Maths now that it is not just all about right and wrong answers.” Pupils ... enjoy their mathematics lessons and feel they are making progress. Formal assessments across the Trust show pupils continue to make progress.”* [E]

Independent evaluation of the MiC and MSM projects highlighted the value of our CPD model, with a key focus on developing understanding of RME philosophy and theory ([Searle & Barmby, 2012](#)). Hough's reputation in RME-related CPD has led to work with the [North-West One Mathematics Hub](#) as a strategic partner since 2014, advising and influencing decision-making. The GCSE resit project has had particular impact, and she has led training of four cohorts of teachers on 'Supporting Post-16 GCSE Resit', (approx. 80 teachers in the NW1 Hub and approx. 100 teachers in Southampton and Birmingham). Meanwhile, Gough has led training for five cohorts of teachers on the Hub-based Teacher Subject Specialism Training (TSST), training 60 teachers - new to mathematics teaching - in RME methods. North-West One Maths Hub lead Simon Mazumder writes: *“The work NW1 has carried out with MMU and the RME project has resulted in many innovation work streams that have been currently adopted as National Collaborative Projects ... Sue and other MMU colleagues have not only helped design but also facilitate on these work streams. Currently over 600 colleagues in both primary and secondary phases have participated. The result of this work has been a greater cohesion of the school's community in Greater Manchester and the ability for NW1 to meet its community's needs. The popularity and impact have been significant and looking at social mobility index the maths work carried out has improved child mobility and raised standards across all phases particularly primary.”* [F]

Recognition of Manchester Metropolitan's work in RME and its potential for deepening understanding of mathematics has led to invitations from policy and practice bodies to develop materials, influence assessment, contribute to curriculum development and train teachers/CPD leads in order to enhance student achievement. In 2013, the DfE invited Hough to be one of three developers on its [Multiplicative Reasoning \(MR\)](#) project, aiming to develop teacher subject and pedagogic knowledge in MR. Hough was responsible for writing resources (one third of the MR resources are based on RME, via Hough's input) and training CPD leads for rolling out the programme nationwide. Evaluation ([DfE, 2015](#)) reported a positive impact on students' relationships with mathematics [F]. In the NW1 Hub, MR has been delivered to five cohorts, involving approx. 100 teachers. Hough served on the Teaching [Core Maths](#) Advisory Board in 2015, training CPD leads in four national centres during 2017/18, reaching approx. 130 teachers. Eade, Dickinson, Hough and Gough jointly trained 120 teachers in seven locations across the country in 2018/19 in Core Maths 'Keeping the Context Alive' days. In 2016, the AQA invited Hough to sit on its expert panel on the basis of her expertise in RME, feeding into assessment design. Her input has led to recognition in marking schemes of non-routine visualisations of question solutions. Her post-16 work led the [Institute of Mathematics and its Applications](#) to invite her to join its Schools and Further Education committee in 2018, commenting on, and making, policy recommendations. Both Hough and Solomon work with the recently-formed [Centres for Excellence in Mathematics](#) (CFEM), formed to address problems in post-16 mathematics education. One element of its strategy is to work with contextualisation, drawing on RME's track record in this area. Manchester Metropolitan's work on developing and researching RME is described on a permanent page on the influential [Mathematics in Education and Industry \(MEI\) website](#). Stella Dudzic, MEI Programme Leader (Curriculum and Resources) writes: *“We make use of the ideas we have learnt from the team ... in the teaching resources we write and in the professional development we offer. ... Videos of the team at MMU ... available on [MEI's YouTube channel](#) ... are in the top most watched videos on the channel. The most popular video ... has 3,500 views. ... MEI has developed a contextualisation toolkit for teachers*

of post-16 GCSE Mathematics; our thinking ... has been influenced by our work with the Mathematics Education team at MMU ... Our experience of working with RME continues to influence the strategies we include in our professional development ...” [G]

Manchester Metropolitan’s work on RME has also had international impact. In 2011, the Ministry of Education in the Cayman Islands invited Eade to work as numeracy specialist on the primary curriculum for mathematics. Over the course of seven years, he integrated scenarios, developed during the MiC project, into the curriculum and developed a ‘Maths Recovery’ teacher-training programme (60+ teachers), influencing teachers’ practice and student attainment [1]. Resources are based on RME principles of use of context and problem-solving. Student achievement rose from 25% at expected level and 5% above in 2011, to 62% at expected level and 25% above in 2018 [1, H].

In July 2019, Hough, Gough, Kathotia and Solomon visited the [Vidya Bhawan Society](#) in Rajasthan, India. The Society is committed to ‘Social Transformation Through Education’, and saw the benefits of developing RME-based mathematics in local schools as a means of raising achievement and retention among disadvantaged students. The team realised that access to mathematical reasoning and literacy materials could benefit this group, and in February 2020, Kathotia and Solomon returned to India to work on supporting the local team of five educational resource workers to expand their understanding of mathematics teaching and learning, and to evaluate their own materials development and implementation. Support for this work continued online during the Covid-19 pandemic [I].

RME’s potential for inclusive mathematics classrooms underpins the Norwegian Research Council-funded IMaT project, which includes an RME-based CPD intervention, in collaboration with Manchester Metropolitan. The OsloMet Kompetanse for Kvalitet (‘Competence for Quality’) course is delivered to 160 teachers each year, with 40 teachers taking part in the IMaT project with access to Manchester Metropolitan materials [J].

5. Sources to corroborate the impact

[A] Feedback from EEF project schools on roll-out beyond project.

[B] Testimonial, Head of Mathematics, Hadlow Rural School, Kent (EEF project school).

[C] i EEF project COVID-19 extension plan; ii EEF project on-line training attendance.

[D] Sales statement, Hodder Books.

[E] i Kemp, A. ‘[We’re top of Stockport’s state school GCSE league](#)’, Cheadle Hulme High School, 23 August 2018; ii Curriculum Description (Maths), Cheadle Hulme High School; iii Testimonial, Director of Maths, Laurus Trust.

[F] i NW1 Maths Hub profiles (Sue Hough, p6); ii Testimonial, Maths Hub Lead (NW1 Hub); iii DfE, [Multiplicative reasoning professional development programme: research report](#) (commissioned evaluation), June 2015.

[G] i Evidence of expert panel roles – Hough; ii MEI, [RME webpage](#); iii Testimonial, Curriculum Programme Leader, MEI.

[H] i Caymans Ministry of Education, Strategy for Primary Mathematics; ii Sample lessons and [resources](#), Cayman Islands Mathematics Curriculum; iii Statement of impact on student achievement, Cayman Islands, Senior Policy Advisor, Cayman Islands Government (independent capacity); iv Cayman Compass, ‘[Math intervention boosts performance scores](#)’, 1 May 2018.

[I] Report on RME-related activities delivered in collaboration with Vidya Bhawan Society.

[J] Oslo Met, IMaT – [Inclusive Mathematics Teaching project webpage](#).