


<b>Section A</b>		
<b>Institution:</b> UoA 10: University of St Andrews		
<b>Unit of Assessment:</b> Mathematical Sciences		
<b>Title of case study:</b> Bringing Mathematics and its History to diverse audiences worldwide		
<b>Period when the underpinning research was undertaken:</b> 2000 - 31 December 2020		
<b>Details of staff conducting the underpinning research from the submitting unit:</b>		
<b>Name(s):</b>	<b>Role(s) (e.g. job title):</b>	<b>Period(s) employed by submitting HEI:</b>
Alex Craik	Professor	01 October 1963 - 30 September 2003
Edmund Robertson	Professor	01 October 1968 - 30 September 2008
John O'Connor	Senior Lecturer	01 October 1970 - 30 September 2010
Isobel Falconer	Reader	01 October 2015 - present
<b>Period when the claimed impact occurred:</b> 2014 - 31 December 2020		
<b>Is this case study continued from a case study submitted in 2014?</b> No		
<b>Section B</b>		
<b>1. Summary of the impact</b> (indicative maximum 100 words)		
<p>The MacTutor History of Mathematics Archive at the University of St Andrews is one of the most accessed resources worldwide for mathematics and its history. The website has sustained an average of approximately 2,000,000 hits per week since August 2013. MacTutor includes detailed biographies of approximately 3,000 mathematicians, many of whom do not appear anywhere else. Over 2,000 other pages of supporting material and essays supplement the biographies, which are carefully researched from primary sources to give fresh insights and an authoritative account. MacTutor continues to grow, with new material regularly being researched and added.</p> <p>MacTutor communicates the essence and importance of mathematics, inspiring a broad audience in almost every country in the world. It is widely used in <b>education</b>. University and College courses worldwide draw heavily on MacTutor, as do secondary schools. MacTutor's creators received the London Mathematical Society's Hirst Prize in 2015 for its contribution as a history of mathematics resource for learners and teachers all over the world. It <b>influences the mathematical awareness of individuals</b> through its uptake in Wikipedia and social media, where it has prompted public discussion of mathematics. Since August 2013, MacTutor has stimulated over 170 Wikipedia articles, and the British Society for History of Mathematics' twitter feed provides a daily link to MacTutor's Mathematician of the Day page. Through its extensive use by the <b>science media</b>, MacTutor has influenced professionals and the public. It has been a seminal resource for over 60 popular books, TV and radio broadcasts and lectures.</p>		
<b>2. Underpinning research</b> (indicative maximum 500 words)		
<p>Research in history of mathematics at St Andrews is characterized by a biographical approach. Elsewhere, more common forms are either to examine the development of a mathematical idea or technique through history, or to investigate the culture and social practices of mathematics. Biography brings these two together as they played out in the work of an individual, enabling the interaction with culture to provide new insights into their mathematics. A particular strength is the history of Scottish mathematicians, some previously little known. It draws on the outstanding collection of manuscripts and rare books in the University of St Andrews Special Collections. For example, between 2000 and 2003, Craik investigated the mathematics of William Wallace, John Leslie and Thomas Carlyle [R1], and Edward Sang [R2]; since 2017, Falconer has researched James Clerk Maxwell [R3], Peter Guthrie Tait and William Thomson [R4].</p> <p>William Wallace, Professor of Mathematics at Edinburgh, wrote the first complete English</p>		

language account of the calculus using the differential notation which is now universally adopted. In [R1] Craik counters the previous Cambridge-dominated historiography of early 19<sup>th</sup> century British mathematics by analysing Wallace's role, along with Leslie and Carlyle, in introducing modern analysis to Britain. The Scot, Edward Sang, was little known to historians of mathematics, but Craik asserts that he was one of the great producers of mathematical tables [R2]. This paper was pioneering in its attention to mathematical practices and the role of women and family in mathematical enterprise. Falconer's paper [R3] demonstrates the power of the mathematical drivers that shaped electrical science in a hitherto neglected aspect of the work of James Clerk Maxwell, the great Scottish physicist. Her review paper [R4] is novel in viewing mathematics and physics as an integrated subject in assessing the work of the Scots Maxwell, Tait, and Thomson, in vortex dynamics.

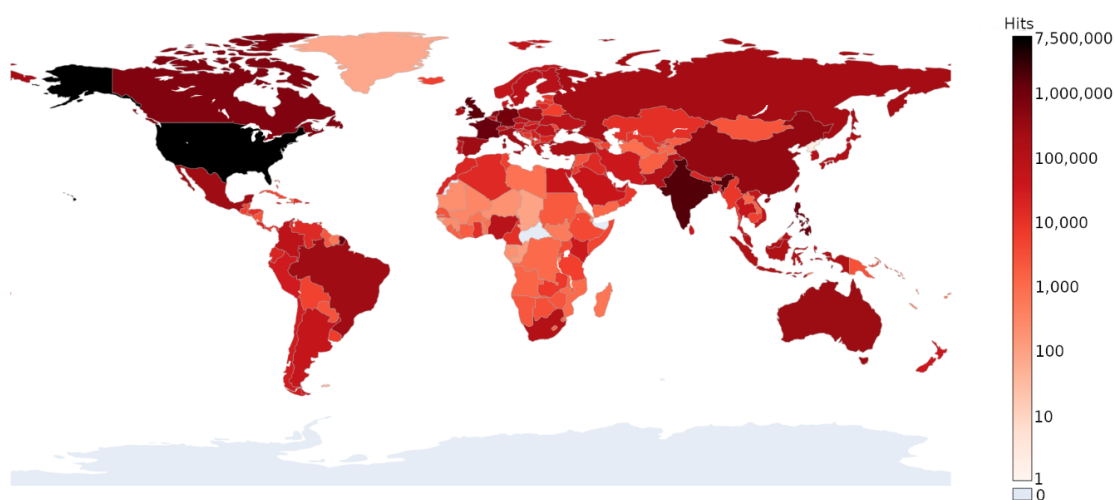
More widely, Robertson and O'Connor published in 2002 on the Irish mathematician William Rowan Hamilton [R5]. On the applied side, Craik's wide-ranging 2004 paper [R6], submitted in 2002, describes the contributions of Continental European and British mathematicians that set the scene for Stokes' innovations in water-wave theory.

### 3. References to the research (indicative maximum of six references)

This is a representative sample of research that is part of a larger body of work of over 30 papers. The research was peer-reviewed and published in highly regarded journals.

- [R1] Craik, A. D. D. Geometry versus analysis in early 19th-century Scotland: John Leslie, William Wallace, and Thomas Carlyle, *Historia Mathematica*, 27 (2000), 133-163. DOI: [10.1006/hmat.1999.2264](https://doi.org/10.1006/hmat.1999.2264)
- [R2] Craik, A. D. D. The logarithmic tables of Edward Sang and his daughters' *Historia Mathematica*, 30 (2003), 47-84. DOI: [10.1016/S0315-0860\(02\)00018-6](https://doi.org/10.1016/S0315-0860(02)00018-6)
- [R3] Falconer, I. J. No actual measurement ... was required: Maxwell and Cavendish's null method for the inverse square law of electrostatics, *Studies in History and Philosophy of Science A* 65-66 suppl. C (2017), 74-86. DOI: [10.1016/j.shpsa.2017.05.001](https://doi.org/10.1016/j.shpsa.2017.05.001)
- [R4] Falconer, I. J. Vortices and Atoms in the Maxwellian Era. *Philosophical Transactions of the Royal Society A*, 377: 20180451 (2019). DOI: [10.1098/rsta.2018.0451](https://doi.org/10.1098/rsta.2018.0451)
- [R5] Robertson, E.F. & O'Connor, J. William Rowan Hamilton 1805-1865, in *Physicists of Ireland: Passion and Precision*, Institute of Physics (2003), 61-68. DOI: [10.1201/9781420033175](https://doi.org/10.1201/9781420033175) (judged a "worthy addition to existing biographical material" by the *British Journal for the History of Science* review <https://www.jstor.org/stable/4028713>)
- [R6] Craik, A. D. D. The origins of water wave theory. *Annual Review of Fluid Mechanics* 36 (2004), 1-28. DOI: [10.1146/annurev.fluid.36.050802.122118](https://doi.org/10.1146/annurev.fluid.36.050802.122118)

### 4. Details of the impact (indicative maximum 750 words)



*Distribution of Hits per Country on MacTutor 25 August – 23 November 2020*

With approximately 2,000,000 people (distinct users), the MacTutor History of Mathematics Archive (<https://mathshistory.st-andrews.ac.uk/>) is ranked 2<sup>nd</sup> in the world in a Google search for

“history AND mathematics”, 2<sup>nd</sup> for “history of mathematics”, and 22<sup>nd</sup> for “mathematics” [S1, pp. 1-8]. Users originate from around the globe as shown in the distribution map above.

The 11,300,000 hits in November 2018 are only slightly above average. Each week, between 1 August 2013 and 31 July 2020, about 1,500,000 pages (over 35Gigabytes of data) were downloaded by approximately 100,000 people (individual users) [S2]. On 20 November 2020, 53,756 people visited the site in a single day [S2, p. 21]. A high proportion of visitors come from outside universities; around 40% of download requests come from .com domains. Most months record hits from nearly every country in the world, with the UK, USA, India, France, and Germany generally the highest users. MacTutor’s uptake is particularly noticeable in three areas: education, the mathematical awareness of individuals, and the science media.

### **Influencing the delivery of curriculum in HEIs and secondary schools world-wide**

MacTutor’s wide recognition as a major educational resource marks its influence on mathematics curricula. In 2015, its originators, Robertson and O’Connor, received the London Mathematical Society’s Hirst Prize. MacTutor was judged as, “*the most widely used and influential web-based resource in history of mathematics*”. It has “*become a hugely successful resource for school-children, undergraduates, graduate students and their teachers all over the world.*” Significantly, “*It bridges the gap between old books and modern journals, and its biographies give lives to names otherwise known only for the theorems to which they are attached.*” [S3, p. 1054].

A Google search gives an indication of how widely educators reference MacTutor in online materials. Searching for “mactutor” in the US educational domain .edu returns over 4,800 results, while “<https://mathshistory.st-andrews.ac.uk/>” (one of MacTutor’s urls until 2019) on .edu returns over 75,400 [S1, pp. 9, 12]. Additionally, HEIs and schools across the world use MacTutor as a core part of their syllabus, as enrichment background reading, or as reference for pupils’ homework projects. The geographical range and types of use are shown by the following representative examples. Between 2019 and 2020, the History of Mathematics module at The York University, Toronto, Canada is heavily based on MacTutor, as the syllabus attests: “*Over the course of the year various readings on the MacTutor page will be assigned; the site will serve as general reference*” [S4, p. 1]. The University of Rome’s course, ‘Corso monografico di Storia della Matematica’, run since November 2013, refers students directly to MacTutor for their project work. In 2016, it advised, “*The quality of the biographies you find on other sites is generally inferior*” [S4, pp. 5, 8]. Similarly, a local-government sponsored study site from Brevard, Florida, exemplifies MacTutor’s relevance to school education when it refers secondary-level pupils to MacTutor [S4, p. 17].

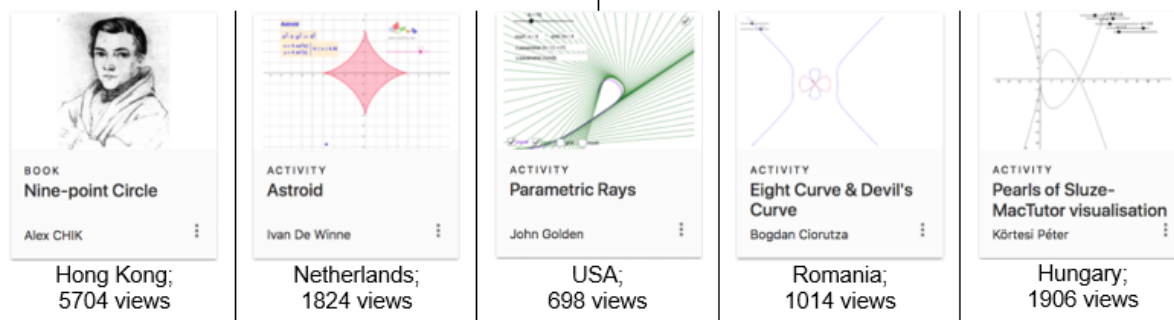
MacTutor’s role in the professional development of mathematics teachers contributes to goals believed to improve the quality of teaching. In 2003, the USA’s National Council of Teachers of Mathematics required teachers to have knowledge of the history of mathematics (NCTM, 2003, *Program standards for initial preparation of mathematics teachers*). Recently, the Royal Society’s Advisory Committee on Mathematics Education recommended that teachers reflect on their understanding of the history of mathematics as part of their professional development (ACME 2016, *Professional learning for all teachers of mathematics*). MacTutor’s contribution to professional development is evident on international online platforms such as GeoGebra and Merlot, where teachers share and discuss new activities they have devised for their pupils. The figure below shows a sample of the new activities based on MacTutor that have been shared on Geogebra since August 2013, along with their countries of origin and viewing statistics; a high proportion of these activities exploit MacTutor’s “Famous Curves” section [S5].

### **Increasing individuals’ awareness and engagement with mathematics and its history**

MacTutor contributes to a stated aim of governments, public and private bodies, of raising public engagement with mathematics. The ‘Stockholm Declaration’ of 2013, for example, calls for an “*understanding of Mathematics by the entire population as a critical right*”<sup>1</sup>. MacTutor counters the deep-rooted negative attitudes to mathematics, which governments perceive as an economic

<sup>1</sup> <https://curriculumredesign.org/wp-content/uploads/Stockholm-Declaration-CCR-FINAL.pdf>

barrier (Marginson et al, 2013, [STEM: country comparisons](#), Australian Council of Learned



*Some of the new activities that use MacTutor that teachers have shared on GeoGebra since August 2013, showing their countries of origin and viewing figures*

Academies). The uptake of MacTutor on web2.0 and social media platforms, described below, demonstrates how it helps to overcome negativity by building bridges between mathematics, mathematicians, and wider culture.

MacTutor is a stimulus to new Wikipedia pages. It has prompted around 150 new Wikipedia biographies since August 2013. Wikipedia's "*Missing encyclopedic articles*" project explicitly called for the creation of pages for mathematicians present in MacTutor but missing from Wikipedia [S6]. Between August 2013 and August 2017, this project saw 40 articles added to Wikipedia. Wikipedia monitors the growth of MacTutor, and in 2017 found 359 further mathematicians added to MacTutor but not yet in Wikipedia. By 31 July 2020, 110 biographies had been written for Wikipedia [S6].

Social media provide additional channels through which MacTutor reaches individuals directly. For example, the twitter feed @mathshistory, maintained by the British Society for History of Mathematics, provides a daily link to MacTutor's Mathematician of the Day page. Between August 2013 and December 2020, its followers increased rapidly from 16,000 to 79,300 people [S7, p. 1]. Hacker News, an international social media news site is responsible for a large number of individual interactions with MacTutor. In August 2020, it was the 5<sup>th</sup> biggest referrer to the site, after search engines such as Google and Bing; it was closely followed by Twitter in 7<sup>th</sup> place and Facebook in 9<sup>th</sup> [S2, p. 22]. A January 2016 discussion on Reddit, another social media news site, exemplifies MacTutor's ability to engage such individuals with mathematical concepts. Prompted by the biography of George Dantzig, the thread ran to an exceptional 501 posts by 318 individuals and was "upvoted" (highly rated) by 90% of its 11,400 readers. It involved those who professed, "*I've never been great at math*" in a debate with the more knowledgeable, that reached considerable depth over the distinction between problems that are unsolved, unsolvable, or undecidable, and concepts of uniqueness and incompleteness [S8, pp. 2-3]. Another Reddit discussion, in December 2018, was prompted by the biography of Verhulst, a pioneer of the logistic function. Debate was heated, but concluded with a post, "*At least people here get a chance to learn about the logistic model, carrying capacity, etc*" [S8, p. 5]. The diverse perspectives from which the public engage with mathematics through MacTutor are illustrated by links added to various websites between August 2013, ranging from a Chinese blog ranking mathematicians, drawing exclusively on MacTutor, and a June 2017 discussion on Hacker News of the chronology of mathematics, to a Russian forum on card gambling problems, and sites devoted to family historians, coin collectors, and the professional development of actuaries.

### **Use of MacTutor influences professionals and the public through science media**

MacTutor enhances public awareness of mathematics indirectly through its extensive use by popular science writers, lecturers and broadcasters. When asked in March 2019, "*What are the 5 math books you would gift to every aspiring female mathematician*", the Mathematics Editor at Princeton University Press, responded, "*My first suggestion is not a book but a wonderful website, MacTutor History of Mathematics... there is a link to Female Mathematicians, which is updated regularly.*" [S9, p. 2]. A best-selling mathematics populariser used MacTutor for specific information on 27 mathematicians for 4 books between 2014 and 2018, published in at least 10 countries [S10, pp. 1-2]. His *Significant Figures* (2017), which used MacTutor for 15 mathematicians, is #25 in Amazon.com's Teen & Young Adult Math eBooks (February 2020)



demonstrating a large and approving younger readership [S10, p. 5]. Between 1 Jan 2014 and 31 July 2020, MacTutor was referenced in at least 60 books, on topics ranging from 'Multiculturalism and the Convergence of Faith and Practical Wisdom in Modern Society' to 'The Trouble with Psychotherapy: Counselling and Common Sense' [S11]. It is a resource for broadcast media, especially Melvyn Bragg's Radio 4 *In Our Time*, with an audience of over 5,500,000 people per episode covering at least 48 countries, which has featured Group members 5 times since 2014; Group member Roney Dougal's episode on P v NP was one of listeners' top 10 of all time [S12]. As a result of MacTutor, the Group receives requests for advice from the media nearly every month.

To ensure the sustainability of its impact, MacTutor is developed continually. The research described and listed in Sections 2 & 3, respectively, is a small sampling of the material that continually informs additions to the site. MacTutor content is produced entirely by members of the University of St Andrews School of Mathematics & Statistics and is built on the School's webserver. The software was upgraded, and the site was migrated to a new platform in Spring 2020, a new, permanent, member of academic staff was appointed in 2020, and the material has been openly licensed for the first time, enabling even wider uptake. Continuing the legacy of MacTutor, academic staff collaborate with emeritus staff, especially Robertson and O'Connor, to develop and augment the research specifically for MacTutor, resulting in 3,000 detailed biographies and 2,000 pages of essays and supporting material that are praised for their quality and the comprehensive bibliographies that accompany them. In 2016, O'Connor added reciprocal links between the entries for 1,706 mathematicians who appear in both MacTutor and MathSciNet, and in 2017 links to Singmaster's *Mathematical Gazetteer of the British Isles*. In 2019 Robertson added 623 African mathematicians, with full biographies of 33, supporting use in developing countries. Additional material is added frequently, with feedback from users and mathematicians contributing to its breadth and efficacy. This ensures that the vitality and relevance of MacTutor continues. As the Spanish blog "Matemáticas y sus fronteras" commented in 2016 on our sustainability plans, "*I hope you succeed in your endeavour, MacTutor is already a world heritage*" [S13, p. 3].

##### **5. Sources to corroborate the impact** (indicative maximum of ten references)

[S1] Google results for relevant domain searches.

[S2] Site statistics from 2017 on, with link to current statistics.

[S3] Hirst Prize citation, *Bulletin of the London Mathematical Society* 47 (2015) 1042–1055, DOI: [10.1112/blms/bdv079](https://doi.org/10.1112/blms/bdv079)

[S4] Pdfs of curriculum sites mentioned.

[S5] Search results on GeoGebra with viewing statistics for articles created since 1 Aug 2013.

[S6] Links and details of the Wikipedia matching process.

[S7] Pdf of @MathsHistory on Twitter.

[S8] Reddit discussions of Dantzig. and of Verhulst.

[S9] Interview with the Mathematics Editor at Princeton University Press.

[S10] Letter from a Mathematics Populariser, evidencing his use of MacTutor, and of Amazon.com rankings of his book *Significant Figures*.

[S11] List of books published since 1 Aug 2013 which reference MacTutor.

[S12] Links to broadcasts featuring history group members since 1 Aug 2013.

[S13] Spanish blog about MacTutor.